

Writing

ARITHMETICK

And

Merchants Accompts,

Or the true Italian method of

BOOK-KEEPING,

Taught by

Ellis Webster,

In Orange Court near

Leicester-Square, LONDON:

Youth Boarded.

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ARITHMETIC

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E P I T O M E :

O R, A

C O M P E N D I U M

O F A L L T H E

R U L E S,

B O T H

V U L G A R and D E C I M A L.

W H E R E I N

CLEAR and PLAIN DEMONSTRATIONS
are deduced from the Principles of *Arithmetic*
itself; without either Reference to *Euclid*, or
Use of *Algebra*.

By W. WEBSTER, Writing-Master.

The SEVENTH EDITION,

Carefully Corrected, with ADDITIONS, by

ELLIS WEBSTER, Writing-Master and Accomptant.

L O N D O N :

Printed for D. BROWNE, at the *Black-Swan* without
Temple-Bar; C. HITCH, at the *Red-Lion* in *Pater-*
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Orange-court, in *Castle-street*, *Leicester-fields*.

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To the RIGHT HONOURABLE

Sir Robert Walpole,

Knight of the Most Noble Order
of the Garter, First Lord Commissioner
of the Treasury, and Chancellor of
the Exchequer, &c.

S I R,

THOUGH the Distrust so natural
to a young Author, on his first
Performance, did indeed discourage me
from complying with my Inclination,
in offering this Attempt to Your Ho-
NOUR on its first publication; yet, being
a little encouraged by the favourable
Reception its first Edition met with in
the World, I did, with all Humility,
presume to lay the second Impression at
your Feet; well knowing the Advantage
it would appear with under the Patron-
age of a NAME so eminent for the En-

A 2 couragement

D E D I C A T I O N.

couragement of *Clerkship*, and of every other Part of useful *Literature*.

Upon the third Revival of this Work for the Press, I made such Additions and Improvements, as have, I hope, given it somewhat a better Title to so great a Protection; but still its Imperfections, I am very sensible, will be but too apparent to Your HONOUR's Penetration: Yet that Goodness, of which I have already had so abundant Experience, gives me Hopes Your HONOUR will both excuse what may remain amiss, and forgive this renewed Presumption of,

Your HONOUR's

Most Obedient, and

Most Humble Servant,

William Webster.

P R E F A C E.

THE Indulgence with which the former Editions of this Treatise have been received, has encouraged me to endeavour the making it still more worthy the Acceptance of the Public; not only by correcting the many Errors which had before escaped, but also by enlarging it with very considerable Additions: Having from the third Impression added the *Theory* to the *Practic Part*, and endeavoured to inform the Reader's Judgment, as well as to assist his Memory.

And this I have attempted in a Manner very different from the ordinary Practice of Writers on this Subject, having never once quoted *Euclid*, nor perplexed the Learner with (to him unintelligible) *Algebraical* Demonstrations.

ARITHMETIC and GEOMETRY are justly looked upon as the two main Pillars of the Mathematics, upon which the whole

P R E F A C E.

Fabric is raised ; but, in comparing them, they are too often confounded : For, how great a Similitude soever there may be betwixt them, in some Respects, yet, as their Subjects are different, so their Principles are certainly distinct, and each may be accounted for without References to the other. Such Comparisons may indeed be useful by way of Illustration, and give great Satisfaction to those who have a competent Knowledge in both ; but Conclusions from *Geometrical* Demonstrations are beyond the reach of the young Arithmetician ; and the Theory of *Lines* and *Diagrams*, as they are commonly made use of, can only serve to bewilder and confound him.

ALGEBRA, indeed, is not properly a Science distinct from Arithmetic, but is only a different Method of Computation, performed by substituting *Letters* in the Place of *Figures*, and expressing the several Parts of the Operation by *Symbolical Characters*. 'Tis true, great Discoveries have been made in Numbers by this Means, and Arithmetic owes many of its Improvements to this admirable Invention : Besides, it furnishes us with a short Way of arguing, and brings the Proofs of a Proposition into a little Compass. But however useful it may be to those who understand it, it is certainly an unin-

P R E F A C E.

unintelligible jargon to the mere Numerist, and can give him very little satisfaction when laid before him as a Demonstration.

Our knowledge in science should rise gradually from one step to another, and our references should be always backwards to what went before; but as the useful parts of Arithmetic have been commonly taught by practical Rules, which are usually taken upon content, so little exactness is generally observed in the order of their delivery, as to their dependance upon each other: The reason of which is plain; for as all Affairs do not require the whole knowledge of Numbers, and as most common Business may be done without much Skill in *Fractions*; so it is not only common, but reasonable enough, to carry the Learner thro' whatever may be performed by *Whole Numbers*, before he is entered upon the greater difficulties of *Fractional Parts*. However, the gradation mentioned should be observed as much as possible, especially in the Business of Demonstration; and the Maxims referred to for explanation should never be of a more abstruse nature than the thing to be explained.

I have therefore endeavoured to give the inquisitive Reader who would understand the reason of things, all the satisfaction I

P R E F A C E.

can ; and have attempted to account for the Principles of Arithmetic from such self-evident Propositions, and natural considerations, as flow most immediately from Numbers themselves, and fall within the comprehension of such who are the common consulters of *Arithmetical Tracts* ; and who, it may be presumed, are generally unacquainted with the more abstruse parts of the Mathematics.

My first design in this Treatise was, according to its Title, to epitomize the Art ; and to reduce to a *Pocket-size*, what has sometimes been swell'd to a *Folio*. Indeed what I first published was no more than a practical Memorandum, and consequently consisted only of Rules and Examples, with few or no explications ; but tho' the enlargements are now very considerable, I have not yet gone beyond my design ; it is still a Compendium, and far short in bulk of other Treatises on the subject. I may therefore now recommend it to the ingenious *Clerks* and *Accomptants* of the several Offices of *Great Britain*, (for whose Use especially it was at first design'd) and to such other *Gentlemen* whose Business requires the practice of Arithmetic, with more confidence than before ; it being very little increas'd in Size, tho' very much improv'd in Precept.

As

P R E F A C E.

As I suppose my Reader acquainted with the manner of performing the common Rules of *Addition*, *Subtraction*, *Multiplication*, and *Division*; so I have made the Work short, by omitting such needless instructions: but the *Reasons* of those operations (being the Fundamentals of Arithmetic) I thought necessary to lay down. The *Rule of Three* I have also endeavoured so to explain, that the Application of *Proportion*, as it runs thro' almost all the other Rules, may be fully understood.

And for those practices of Arithmetic which will admit of no other than an *Algebraical* Demonstration, being generally Parts more curious than useful, such as are some propositions in *Progression*, and the *Double Rule of False*; I have thought it sufficient to give the practical Rules for their performance, without their Demonstration; since those who are proficient in *Algebra* cannot be thought to want such explanation; and those who are not cannot be supposed to understand it.

I have indeed ventured to account for the Extraction of the Roots, in a manner which seems something contradictory to my profess'd design: but when it is consider'd, that the Subject of those Operations is entirely

P R E F A C E.

geometrical, though the Operations themselves are *numerically* performed, it will be found, that there is no other rational method of Demonstration, but that of *Lines*; which, however, I hope, I have so laid down, as to be clear and intelligible to every Reader of common sense, who but understands the difference betwixt a *Line*, a *Surface*, and a *Solid*.

As to the disposition of the *Chapters*, they follow much in the same order as they are generally taught in *Schools*: For tho' I am sensible, that the Rules of *Practice* are not thoroughly to be understood without some knowledge of *Vulgar Fractions*, yet I have chosen to place Fractions in the last Chapter of *Vulgar Arithmetic*, because (as I before observed) most common Business may be done by whole Numbers. So the speculative Rules of *Progression*, or *Arithmetical* and *Geometrical Proportion*, which, in a just *Theoretic* way of treating the subject of Numbers, ought to precede the *Rule of Three*, I have thought better to place just before *Vulgar Fractions*: however, *Progression* and *Vulgar Fractions*, being each distinct Chapters, the Reader may look into them whenever he thinks it most proper.

The

P R E F A C E.

The second Part of this Treatise, which contains the Doctrine of *Decimal Fractions*, &c. is a kind of Arithmetic peculiarly, as it were, adapted to the concerns of *Gentlemen*: I have therefore been more large than ordinary upon that Subject, and have run over the several Rules again, to shew its particular use and application. In the Chapter of *Interest* and *Rebate*, its excellency will most appear, every Calculation of that kind being performed to great advantage by Decimal Numbers. Under that head will likewise be found several useful *Tables* (which, that they might be perfectly correct, are exactly engrav'd on Copper) for the ready discovering the amount, and present worth, both of single Sums, and of Annuities. And as the Interest of Money is now at a lower Rate than when this Treatise was first published, I have in this Impression added Tables at 3 and 4 *per Cent.* to those before published at 5 and 6. There are, besides these, some other necessary *Tables* interspersed through this part of Arithmetic (which are also with the same exactness, printed from Copper-Plates); as those for the reduction of the known parts of Coin, Time, Weight, and Measure, into decimal Parts of their proper *Integers*; and also that most useful *Table*, which shews the number of Days from

P R E F A C E.

any day in any Month to the same day in any other. The *Contents* will shew where to find any of these things; and, in the Work itself, are sufficient explanations both of their formation and use. I shall therefore add nothing further by way of Preface, but submit the whole Performance to the judgment and candour of the Public.

VULGAR

VULGAR ARITHMETIC,

I N

Whole Numbers and Fractions.

AS it is inconsistent with the intended brevity of this Treatise, to enter upon a nice enquiry after the first Inventors of Arithmetic, or by what degrees it has been raised to its present perfection; so neither is it my design, in the following pages, to initiate absolute Beginners; but supposing my reader acquainted, at least, with the methods of Adding, Subtracting, Multiplying, and Dividing, (tho' perhaps ignorant of the reasons thereof) I shall lay before him, in as few words as possible, the reasons of those Fundamentals, and their Application throughout all the varieties of practice.

Arithmetic (which is justly defined *The Art of Reasoning*) has for its subject, Number; and teaches us to give proper answers to all such questions as demand *How many?* For the more ready performing of which, the ten following Characters; or Figures, have been invented and agreed upon by the greatest part of the known World, *viz.*

1 2 3 4 5 6 7 8 9 0

and by the various combinations and repetitions of these ten Characters, may every Number, how great soever, be easily and expeditiously expressed.

Thus much in general: I now proceed to particulars.

CHAP.

C H A P. I.

*Of the five first and fundamental Rules, viz.
Numeration, Addition, Subtraction, Mul-
tiplication, and Division.*

NUMERATION, OR NOTATION.

BY Numeration we learn the different value of Figures, by their different places; and, of consequence, to read or write any Sum, or Number.

The T A B L E.

9	Units.	1
90	Tens.	12
900	Hundreds.	123
9000	Thousands.	1234
90000	X Thousands.	12345
900000	C Thousands.	123456
9000000	Millions.	1234567
90000000	X Millions.	12345678
900000000	C Millions.	123456789

From this Table may be observed :

1. The names of the several places, *viz.* Units, Tens, Hundreds, &c. which proceed (increasing by a ten-fold proportion) from the right-hand to the left.

2. That every Figure hath two values ; one in itself ; the other from the place it stands in. Thus, on the left-side of the table, the figure 9 in the upper line, standing in the unit's place, is only nine ; but in the
second

second line, being removed into the place of Tens, becomes ninety; and in the third line is nine hundred, &c.

3. That tho' a cypher is nothing in itself, yet it gives value to other figures, by removing them into higher places.

All which being very obvious, I proceed to the next Rule.

A D D I T I O N.

BY Addition we find the whole, or total, of two or more parts, or sums.

In setting down the numbers to be added, care must be taken to place every figure in its proper column; that is, Units under Units, Tens under Tens, &c. Then will the reason of the work (the manner of which, I suppose my reader well acquainted with) appear very evident from this undeniable Maxim, *viz. That the whole is equal to all its Parts.* And the method of setting down the total, may easily be accounted for from the nature of Numeration, which explains the different value of places, as they proceed from the right to the left hand: For as 9 is the greatest simple character, or figure; so every number exceeding 9, being compound, must require more places than one to express it. Thus the number 10 can no otherwise be expressed in figures, but by removing the figure 1 into the place of Tens, which is done by supplying the Unit's place with a cypher: And as it is the same with every other column (Ten being still the proportion of increase) consequently, when the sum of any column amounts to 10, or more, the units exceeding, if there be any, or a cypher, if none, must be set under such column; and the ten, or tens, of the amount, carried on as so many units, to the next column on the left.

What

What is here observed, as to carrying the Tens (the proportion of increase) from one column to another in integers, may be as justly applied to the numbers we stop at, in adding sums of different Denominations.

For your greater ease in casting up of Money, learn the following Table.

PENCE-TABLE.

<i>Pence.</i>		<i>s.</i>	<i>d.</i>		<i>Pence.</i>		<i>s.</i>	<i>d.</i>
20	} is	1	8		80	} is	6	8
30		2	6		90		7	6
40		3	4		100		8	4
50		4	2		110		9	2
60		5	0		120		10	0
70		5	10					

Examples in Whole Numbers, and Money.

<i>l.</i>	<i>Yards.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>
756	1325	735	18	09 $\frac{1}{2}$
132	4532	423	10	10 $\frac{1}{2}$
458	7345	784	12	05 $\frac{3}{4}$
736	1298	297	08	04
857	8473	542	11	11 $\frac{1}{4}$
241	5249	298	14	07 $\frac{3}{4}$
3180	28222	3082	17	00 $\frac{3}{4}$ <i>Total.</i>

Examples

Examples in Avoirdupoise and Troy Weights.

AVOIRDUPOISE.

<i>Tons.</i>	<i>C.</i>	<i>q.</i>	<i>lb.</i>	<i>oz.</i>	<i>drams.</i>	
753	19	3	27	15	15	
347	08	1	17	10	06	
283	11	0	12	03	10	
549	05	2	13	09	07	
251	13	0	15	03	11	
2185	18	1	02	11	01	<i>Total.</i>

TROY-WEIGHT.

<i>lb.</i>	<i>oz.</i>	<i>dwt.</i>	<i>grs.</i>	
327	11	19	23	
429	10	17	19	
274	08	13	09	
478	10	12	05	
326	09	18	11	
1838	04	01	19	<i>Total.</i>
1510	04	01	20	{ <i>Tot. without the Top-line.</i>
1838	04	01	19	
				<i>Proof.</i>

To know how many Drams make an Ounce, or Grains a Penny-weight, &c. see the following Tables, Pages 18.

Proof of Addition.

The proof of this Rule is usually by a second addition, without the top-line ; which second sum, if, when added to the uppermost line, it makes the first total, the work is supposed right. *See the last Example.*

But

But it is as good, if not a better Proof, to produce the same total, by adding your columns both up and down.

SUBTRACTION.

SUBtraction teaches us, by taking a lesser number from a greater, to find the Remainder.

The reason of this Rule is evident from the same principles as Addition; Subtraction being but the reverse thereof: And the number borrowed in any column (like what we stopt at in Addition) being always so many as would make 1 in the next.

Examples in Integers and Money.

<i>Yards.</i>		<i>l.</i>	<i>s.</i>	<i>d.</i>
7146325		<i>Lent</i> 812	13	08 $\frac{1}{4}$
1483972		<i>Paid</i> 190	19	10 $\frac{1}{2}$
<u>Rem. 5662353</u>		<u>Rem. 621</u>	13	09 $\frac{1}{4}$
<u>Proof. 7146325</u>		<u>Proof. 812</u>	13	08 $\frac{1}{4}$

	<i>l.</i>	<i>s.</i>	<i>d.</i>
<i>Borrowed</i>	429	11	08 $\frac{1}{2}$
<i>at several</i>	212	17	10
<i>Times.</i>	356	17	05 $\frac{3}{4}$
<u><i>Borrowed in all</i></u>	<u>999</u>	<u>07</u>	<u>00$\frac{1}{4}$</u>
<i>Paid</i>	519	18	09 $\frac{3}{4}$
<u><i>Rem.</i></u>	<u>479</u>	<u>08</u>	<u>02$\frac{1}{2}$</u>
<u><i>Proof.</i></u>	<u>999</u>	<u>07</u>	<u>00$\frac{1}{4}$</u>

AVOIR.

AVOIRDUPOISE.

<i>Tons.</i>	<i>C.</i>	<i>q.</i>	<i>lb.</i>	<i>oz.</i>	<i>Drams.</i>
92	10	0	07	03	13
14	13	3	11	12	05
77	16	0	23	07	08 <i>Rem.</i>
92	10	0	07	03	13 <i>Proof.</i>

TROY-WEIGHT.

	<i>l.</i>	<i>oz.</i>	<i>d-wts.</i>	<i>grs.</i>
<i>From</i>	672	10	05	09
<i>Take</i>	139	11	05	21
	532	10	19	12 <i>Rem.</i>
	672	10	05	09 <i>Proof.</i>

Proof of Subtraction.

Sums in this Rule are easily proved, by adding their remainders to their lesser numbers; which (if right) will make the greater.

MULTIPLICATION.

Multiplication (which is the fourth Rule) serves instead of many Additions; the product of a Multiplication being only the repetition of the Multiplicand so many times as there are units in the Multiplier.

Note, The ready performance of this and the next Rule, intirely depends upon the perfect knowledge of the following Table.

The

The TABLE.

3 times	{	3	9	7 times	{	7	49
		4	12			8	56
		5	15			9	63
		6	18				
		7	21	8 times	{	8	64
		8	24			9	72
		9	27				
				9 times		9	81
				12 times	{	2	24
4 times	{	4	16			3	36
		5	20			4	48
		6	24			5	60
		7	28			6	72
		8	32			7	84
5 times	{	5	25			8	96
		6	30			9	108
		7	35			10	120
		8	40			11	132
		9	45			12	144
6 times	{	6	36				
		7	42				
		8	48				
		9	54				

Three terms are used in Multiplication, *viz.* *Multiplicand*, *Multiplier*, and *Product*.

The following Example shews to which line each term belongs.

EXAMPLE.

E X A M P L E.

7563254138 *Multiplicand.*

23456789 *Multiplier.*

68069287242	
60506033104	
52942778966	
45379524828	I
37816270690	8+8 <i>Proof.</i>
30253016552	I
22689762414	
15126508276	
177409656468442882 <i>Product.</i>	

That Multiplication serves instead of many Additions, and, consequently, that the truth of its operation depends upon the same Reasons, is easily proved: For suppose it were required to know the sum of four sevens; by Addition, the work will stand thus:

$$\begin{array}{r}
 7 \\
 7 \\
 7 \\
 7 \\
 \hline
 28
 \end{array}$$

Too tedious a method for practice, being discoverable at once, by the Table of Multiplication, to be 28.

Nothing therefore remains to be accounted for, in this Rule, but the reason of placing every following particular product a place nearer to the left-hand than the product foregoing: But Regard being still had to the nature of Numeration, it will plainly appear, that as we begin with the right-hand figure of the Multiplier, or Units place; so the second figure standing in the place of Tens, the product thence arising, must, of

con-

consequence, be ten times greater than the same figure, standing in the place of Units, would have produced ; and so on, applying the same consideration to the produce of all the other figures in the Multiplier.

Proofs of Multiplication.

Multiplication is usually thus proved. Cast out the *nines* from the Multiplicand and Multiplier, and place the remainders on the right and left sides of a Cross, thus made $+$. These two figures multiplied together, must have the *nines* cast out of their product, and the remainder placed at top : Then casting the *nines* also out of the product of your Multiplication, place its remainder at the bottom ; which if it agrees with the figure at the top, the work is supposed right. *See the Example.*

A more certain Proof.

A Multiplication sum is then right, when the product, divided by the Multiplier, quotes the Multiplicand ; or, divided by the Multiplicand, quotes the Multiplier.

ABBREVIATIONS.

(1st.) When either your Multiplicand, or Multiplier, or both, have one or more cyphers to the right-hand, only multiply by the significant figures, and set on the right-hand of the product so many cyphers as were in both the Multiplicand and Multiplier.

EXAMPLES.

$$\begin{array}{r} 527000 \\ 4000 \\ \hline 2108000000 \end{array}$$

$$\begin{array}{r} 7583 \\ 7000 \\ \hline 53081000 \end{array}$$

(2^{dly}.) Therefore any number may be multiplied by 10, 100, 1000, &c. only by placing on the right-hand
of

[11]

of it, one, two, three, or more cyphers : Thus 7295 multiplied by 10, is 72950 ; by 100, is 729500, &c.

(3dly.) To multiply any number by 5, add a cypher to it, and halve it. Or by 15, the same ; and add both lines together.

E X A M P L E S.

5334 multiplied	
by 5 — 53340	by 15 — 53340
<u> </u>	<u>26670</u>
is — 26670	is — 80010

(4thly.) Any number may be multiplied by 11, 111, or 112, &c. as in the following Examples.

E X A M P L E S.

7136 multiplied by 11	by 111 thus: 7136
7136	7136
<u> </u>	7136
78496	<u> </u>
	792096

And by 112, thus: 7136	Or thus: 7136
7136	112
7136	<u> </u>
7136	85632
<u> </u>	7136
799232	<u> </u>
	799232

We now proceed to our fifth Rule.

DIVISION.

BY Division (which is the reverse of Multiplication) we find how often one number is contained in another.

In this rule there are also four terms, the *Divisor*, *Dividend*, *Quotient*, and *Remainder*.

EXAMPLE.

Divisor. *Dividend.* *Quotient.*
 3456789)567895436783(164284

22221653

14809196

9820407

29068298

14139863

312707 *Remainder.*

As Multiplication serves instead of many Additions ; so Division supplies the place of many Subtractions ; as may be thus made evident. Suppose it were required to divide 28 by 7, that is, to find how often 7 is contained in 28 ; by Subtraction the work will stand thus :

28

7

—

21

7

—

14

7

—

7

7

—

By

By which we find that 7 is 4 times contained in the number 28. But it may be discovered by the rule of Division at one trial.

There are many methods of working this rule. The following Example is divided six several ways.

First.

1008(1

42645(1332

32222

333

Third.

1008(1

32)42645(1332

Second.

1008(1

32)42645(1332

32664

996

21

Fourth.

1008(1

32)42645(1332

32

10

10

32)42645(1332

32

96

96

64

Sixth.

32)42645(1332

106

104

85

21

Fifth.

32)42645(1332

32

106

96

104

96

85

64

21

21

Note,

Note, The two last, called the *Italian* ways, are most generally used.

Mr. *Alingham* has, indeed, set down nine ways; but his three others are so very like the 1st, 2d, and 3d, of these, that they can scarce be called different.

ABBREVIATIONS.

(1st.) If there are any cyphers on the right-hand of your Divisor, you may cut off so many cyphers, or figures, on the right-hand of your Dividend; but remember to bring them down (if figures) to the remainder.

EXAMPLE.

21|00)8645|29(411

84

—
24

21

—
35

21

—
1429

(2^{dly}.) By the foregoing rule, you may observe, that to divide by 10, 100, 1000, &c. is only to cut so many figures from the right-hand of the Dividend, as there are cyphers in the Divisor.

EXAMPLE.

1|000)43682|735)

So the Quotient is 43682, the Remainder 735.

(3^{dly}.) When your Divisor is 12, or consists only of one single figure, or can be reduced to one, by cutting off cyphers from its right-hand, the work may be easily performed in one line, thus:

R U L E

R U L E.

Drawing a line under the Dividend, set down under its first figure, how often the Divisor is contained in it; what remains imagine placed before the next figure; and, considering how often your Divisor is contained in the sum it makes, set down the number underneath, as before; and so proceeding through all the figures, set down what remains at last, in the Place where your Quotient used to stand.

E X A M P L E S.

$$4 \overline{)93645(1}$$

$$23411$$

$$12 \overline{)83675(11}$$

$$6972$$

$$7 \overline{)005635(15(}$$

$$805$$

If you are to divide several Numbers by one common Divisor (as in the calculating of Tables, &c.) that you may know exactly at once how often your Divisor will go, in some convenient Corner make a Table of your Divisor, by multiplying it severally by all the nine Digits: Thus, suppose 562 your Divisor:

562	1
1124	2
1686	3
2248	4
2810	5
3372	6
3934	7
4496	8
5058	9

Proofs of Division.

(1st.) Multiplication and Division mutually prove each other: For as if you divide the Product of a Multiplication by the Multiplier, the Quotient will be the Multiplicand; so, if you multiply the Quotient of

a Division by the Divisor, (taking in the remainder) the product will be the Dividend.

(2dly.) Another proof of Division is, by adding together those lines in the following example, marked with Asterisks (being the particular products of the Divisor, multiplied severally by each figure in the Quotient, together with the remainder of the Division) the total of which (if right) will be the Dividend.

(3dly.) Division may also be proved as Multiplication, by a cross, thus; casting out the nines from the Divisor and Quotient, place the remainders on its right and left sides; then multiplying the two figures so placed together, and casting the nines from the product, add what's left to the remainder of the Division; and still casting out the nines, let the overplus be placed at the top; then also casting the nines from the Dividend, set down the figure remaining at the bottom, which if it agrees with that at top, the work may be supposed right. See each proof in the following example.

E X A M P L E.

	736)863256(	1172
	736*	736
3	1272	7032
7+2	736*	3516
3		8204
3d Proof.	5365	
	5152*	862592
		664 Remainder.
	2136	
	1472*	863256 1st Proof.
	664*	
	863256 2d Proof.	

Before we proceed to *Reduction*, it will be proper to insert the following Tables.

TABLES

TABLES of English Coins, Weights, and Measures.

First, of COINS.

ACCOMPTS are kept in Pounds, Shillings, Pence,
and Farthings, thus divided :

4 Farthings	}	make	{	1 Penny,	}	thus marked	{	d.
12 Pence				1 Shilling,				s.
20 Shillings				1 Pound,				l.

But the usual Coins are,

			l.	s.	d.		
Of Gold,	{	A Jacobus,	Value	{	1 5 0		
		A Carolus,			1 3 0		
		A Guinea,			1 1 0		
		A $\frac{1}{2}$ Guinea,			0 10 6		
		A Crown,			0 5 0		
Of Silver,	{	A $\frac{1}{2}$ Crown,	Value	{	0 2 6		
		The names of the rest speak their value, as a Shilling, a Six-pence, a Groat,					
		or 4 d. a Three-pence, a Two pence,					
		a Penny.					
Of Copper,	{	A $\frac{1}{2}$ Penny,	thus writ	{	0 0 0 $\frac{1}{2}$		
		A Farthing,			0 0 0 $\frac{1}{4}$		

Besides the above-mentioned, we have still in use the names of some other pieces, which are now but imaginary, viz.

			l.	s.	d.
A Mark, An Angel, A Noble,	{	Value	{		0 13 4
					0 10 0
					0 6 8

WEIGHTS.

TROY.

24 Grains }
20 Pennywts. } make { 1 Pennywt. } thus { dwts.
12 Ounces } { 1 Ounce, } mark'd { oz.
 } { 1 Pound, } { ℥.

By *Troy Weight* are weighed Jewels, Gold, Silver,
and all Liquors.

A P O T H E C A R I E S.

20 Grains }
3 Scruples } make { 1 Scruple,
8 Drams } { 1 Dram,
12 Ounces } { 1 Ounce,
 } { 1 Pound, } thus mark'd {

By these Weights, Apothecaries compound their Medicines; but buy and sell their Drugs by *Avoir-du-poise*.

AVOIR DU POISE.

16 Drams	} make	1 Ounce,	} thus mark'd.	oz.
16 Ounces		1 Pound,		lb.
28 Pounds		1 quarter of a Hund.		qrs.
4 Quarters		1 Hundred,		C.
20 Hundred		1 Ton,		ton.

This is, at present, the common weight of *England*, by which, Butter, Cheese, and all Groceries, &c. are weighed.

Note, One pound Avoirdupoise, is equal to 14 oz. 11 dwts. 15 grs. $\frac{1}{2}$ Troy; and one ounce Troy is equal to 1 oz. 1 dram, and something above $\frac{1}{2}$, Avoirdupoise.

W O O L-

W O O L - W E I G H T.

7 Pounds	}	make	1 Clove,	}	mark'd as written.
2 Cloves			1 Stone,		
2 Stones			1 Todd,		
6 Todds and $\frac{1}{2}$			1 Wey,		
2 Weys			1 Sack,		
12 Sacks			1 Last,		

L E A D.

C.
19 $\frac{1}{2}$ make a Fodder.

M E A S U R E S.

W I N E.

2 Pints	}	make	1 Quart,	}	thus mark'd	2rt.
4 Quarts			1 Gallon,			Gall.
63 Gallons			1 Hogshead,			Hdd.
2 Hogsheads			1 Pipe,			Pipe.
2 Pipes			1 Tun,			Tun.

B E E R and A L E.

2 Pints	}	make	1 Quart,	}	thus mark'd	2rt.
4 Quarts			1 Gallon,			Gall.
9 Gallons			1 Firkin,			Firk.
2 Firkins			1 Kilderkin,			Kild.
2 Kilderkins			1 Barrel,			Bar.
3 Barrels			1 Butt,			Butt.

Note, 8 Gallons make 1 Firkin of Ale.

D R Y.

8 Pints	}	make	1 Gallon,	}	thus mark'd	1 Gall.
2 Gallons			1 Peck,			1 Peck.
4 Pecks			1 Bushel,			1 Bush.
4 Bushels			1 Coom,			1 Coom.
2 Cooms			1 Quarter,			2r.
5 Quarters			1 Wey,			1 Wey.
2 Weys			1 Last,			1 Last.

L O N G.

3 Barley-Corns	}	make	1 Inch,	}	mark'd as writ.	* 1760 Yards make a Mile.
12 Inches			1 Foot,			
3 Feet			1 Yard*,			
5 Yards and $\frac{1}{2}$			1 Pole or Perch,			
40 Poles			1 Furlong,			
8 Furlongs			1 Mile,			

L A N D.

40 Square Perches	}	make	1 Rood.
4 Roods			1 Acre.

Note, That a Geometrical Pace is 5 Feet, and that there are 1056 such paces in an *English* Mile.

C L O T H.

4 Nails	}	make	1 Quarter,	}	mark'd	1 qr.
4 Quarters			1 Yard,			1 yd.

Note, An *Ell-Flemish* is 3 qrs. of a Yard; an *Ell-English* is 5.

12	}	make	1 Dozen.
12 Dozen			1 Small Gross.
12 Small			1 Great Gross.

CHAP. II.

OF REDUCTION.

Reduction is but an application of Multiplication and Division : For,

First, All great names are brought into small, by multiplying with so many of the little ones as make one of the great.

Secondly, All small names are brought into great, by dividing by so many of the little ones as make one of the great.

Thirdly, To change one sort of money, or weight, &c. into another, is only to bring both into one name, and to divide the one by the other.

EXAMPLES of each Sort.

(First Sort.)

l. s. d.

In 4295 12 3 how many Farthings ?

'tis multiplied by 20, because 20 s. make 1 l. Note, The
 ——— 12 s. are taken in.

85912

'tis multiplied by 12, because 12 d. make 1 s. And here
 ——— the 3 d. are added.

1030947

'tis multiplied by 4, because 4 Farthings make 1 Penny.

Answer, 4123788 Farthings.

(Second Sort.)

In 4123788 Farthings, how many Pounds?

$$\begin{array}{r} 4 \overline{) 4123788} \end{array}$$

$$\begin{array}{r} 12 \overline{) 1030947} 3 \end{array}$$

$$\begin{array}{r} 20 \overline{) 85912} \end{array}$$

4295 l. 12s. 3d. facit.

We divide here by 4, 12, and 20, for the same reasons that we multiplied by them in the first example.

(Third Sort.) s. d.

Change 45967 French Crowns, at 4 6 each, into Guineas

Six-pences in a French Crown. } 9

$\begin{array}{r} 21 \\ 2 \\ \hline \end{array}$	$\begin{array}{r} 45967 \\ 9 \\ \hline \end{array}$
$42 \left\{ \begin{array}{l} \text{Six-pences in} \\ \text{a Guinea.} \end{array} \right.$	$\begin{array}{r} 42 \overline{) 413703} 9850 \text{ Guineas.} \\ 378 \\ \hline 357 \\ 336 \\ \hline 210 \\ 210 \\ \hline \end{array}$

03 Six pences.

More EXAMPLES.

In 85647 Guineas, how many Pounds Sterling?

Answer, 89929 l. 7s.

In

In 59463 Marks, at 13s. 4d. how many Pounds?

Answer, 39642 l.

How many Dollars at 4s. 4d. are there in 89573 Pistoles, at 17s. 6d?

Answer, 361737 Dollars, and 6d. remain.

In 82 lb. 9 oz. 10 dwts. 5 grs. how many Grains?

Answer, 476885 Grains.

In 56 Tons, 13 C. 2 qrs. 21 lb. 7 oz. 11 Drums, how many Drums?

Answer, 32505211 Drums.

How many minutes since the birth of our Saviour, it being 1746 years?

Answer, 917697600 minutes.

How many Gallons are there in 7569 Tuns?

Answer, 1907388 Gallons.

Note, These questions reversed, will make as many more good examples.

C H A P. III.

Of PROPORTION, or the RULE of THREE,
Direct, Indirect, and Double.

THIS Rule is called the RULE of THREE, because by three numbers given we find a fourth number sought; which, when the Proportion is direct, must always bear the same ratio, or proportion, to the third number, as the second bears to the first.

The chief difficulty of this Rule lies in stating its questions. For your direction therefore observe, that, of the three given numbers, two always contain a Supposition, and the third a Demand.

The number then on which the Demand lies, must always be the third in your stating; of the other two, you will be sure to find one of the same quality with the said third; which being made your first, the number left, must, of consequence, fall in the second place, which second number will also always be of the same kind with the fourth, or number sought.

The question being thus stated, you must (if they are not already so) bring your first and third numbers into one name, and your second (if of several denominations) into its lowest term; then multiplying your second and third numbers together, and dividing the product by your first, the quotient will be the answer of your question, in the same denomination you left your second number. *See the Examples.*

E X A M P L E S.

(*Ex. 1st.*) How many Ounces may I buy for 8*l.* if I give after the rate of 2*l.* for 18 Ounces?

1. oz.
If 2 buy—18—what will 8?

2) 144

Answer, 72 oz.

(Ex. 2d.) What must I give for 112 Ells of Holland, if I pay after the rate of 34^l. for 138 Ells?

Ells. l. Ells.
If 138 cost—34—what will 122?

34

488

366

l. s. d.

138) 4148 (30 1 1³/₄

414

8

20

138) 160 (1

138

22

12

138) 264 (1

138

126

4

138) 504 (3

414

90

The Remainder 8, being Parts of a Pound, are multiplied by 20, to see what Shillings they will produce; and so the other Remainders by 12, and 4, to bring out the odd Pence and Farthings.

This way of valuing the Remainder, is more fully explained in the seventh sort of Reduction of Vulgar Fractions.

(Ex. 3d.)

(Ex. 3d.) How many Yards of Muffin can I buy for 42*l.* 12*s.* if 2½ Yards come to 19*s.* 6*d.*?

s. d.	Yards.	l. s.
If 19 6 buy	2½	what will 42 12
2	2	20
39	5	852
		2
		1704
		5
		39)8520(218
		78
		72
		39
		330
		312
		18

2)218(

Answer, 109 Yards.

P R O O F.

Of every four numbers in direct proportion, the product of the two means multiplied into each other, will be equal to the product of the two extremes so multiplied; therefore multiplying your first by your fourth number found, and comparing it with the produce of your second by your third, if the products agree, the work is right. Thus, in the first example, 72 (the fourth number found) multiplied by 2 (the first number) gives 144, equal to the produce of 8 times 18, the second and third numbers.

And from hence arises the invention of the foregoing rule, for finding the fourth number; for if the second number, multiplied by the third, be equal to the

the first multiplied by the fourth, it is plain that if the product of the second and third be divided by the first, the Quotient must be the said fourth number; because every Dividend must be equal to the produce of its Divisor and Quotient, the Remainder (if any) being also considered.

Sums in this rule may also be proved by a back; stating; thus reversing the third Example,

<i>l.</i>	<i>s.</i>	<i>yards.</i>	<i>s.</i>	<i>d.</i>	<i>yards.</i>
If 42	12	will buy 109,	then 19	6	will buy 2½
20		39	2		
852		981	39		
2		327			
1704		1704)4251(2½ yards.			
		3408			
		843			
		2			
		1686			
		18	Remainder of the other		
			stating added!		
		1704)1704(1			
		0			

Though, in the former of the foregoing proofs, the manner of working the *Rule of Three* is sufficiently accounted for, and the reason of the operation plainly enough laid down; yet, as this may justly be looked upon as the main Rule of Arithmetic, and as the same application of proportion runs through almost all its other branches, it may, perhaps, help to make it still more evident, to look back again upon the first example, and observe it in another light. The question there stated was, If 2 *l.*—18 *oz.*—8 *l.* Now had it been, If 1 *l.*—18 *oz.*—8 *l.* 'tis clear, that if 1 *l.* would buy

buy 18 oz. then 8 L . would buy 8 times as many, that is, 144 oz. But then, as this is upon the supposition that 1 L . is the price of 18 oz. so if the price of the 18 oz. is doubled, that is, made 2 L . 'tis plain 144 oz. will then be twice the quantity that 8 L . will buy, and must therefore be divided by 2, to give the true answer. Again, had the supposed price of 18 oz. been 4 L . then would 8 L . have bought but a fourth part of 144 oz. viz. 36 oz. consequently, in what proportion soever the first number increases, in the same proportion must the fourth number decrease; which is plainly effected by dividing by the first number.

Note, In the operation of the *Rule of Three*, the first and third numbers, after preparation, are no more regarded, as of any denomination; but are multiplied and divided with, only as absolute numbers, increasing and decreasing the middle number in just proportion.

Note also, That what has been said of the reason of working the *Rule of Three*, ought also to be considered in the operations of *Interest*, *Rebate*, *Fellowship*, *Exchange*, and all other applications of proportion.

We now proceed to some other practical observations.

1. If you would know at what rate you must sell out your goods by retail, so as to make a proposed gain by the whole, add the money you would gain to the sum the whole goods cost you; and then state your question thus: *If the whole be sold for the total of the Cost and Gain, what must any part?*

E X A M P L E.

Suppose I would, by the selling of 32 yards of Broad Cloth, which cost me 40 L . gain 5 L . for what must I sell it per yard? Then,

yards. *l.* *yard.*
If 32 be sold for 45, what will 1?

<i>l.</i>	<i>l. s. d.</i>
40 Cost.	—
5 Gain.	32)45(1 8 1½ <i>facit.</i>
—	32
45 Total.	—
	13
	20
	—
	32)260(8
	256
	—
	4
	12
	—
	32)48(1
	32
	—
	16
	—
	4
	—
	32)64(2
	64
	—
	0

(2d.) Or if damage having happened to the Cloth, 5 *l.* were to have been lost by the whole, then the said 5 *l.* must have been subtracted from the cost, and the remainder made the second number, as before.

(3d.) If you would barter or exchange your goods for others, first find the value of your own, and then see what quantity of the others the sum will purchase.

EXAMPLES.

EXAMPLES.

What quantity of Pepper, at 3s. 6d. per lb. may I have in exchange for 426 C. of Tobacco, at 53s. per C?

C.	s.	C.
First, If 1	53	426
		53
		—
		1278
		2130
		—

Answer, 22578 Shillings.

s. d.	lb.	s.
Then, If 3 6	1	22578
2		2
—		—
7		7)45156(6
		—

Answer, 6450 Pounds.

ABBREVIATION.

The work of some statings may be much shortened, by dividing their first and second, or first and third numbers, by any figure, so that nothing may remain.

EXAMPLE.

[31]

EXAMPLE.

l. *Man,* *l.*
If 19200 would pay 500 how many would 17664 pay?

$$\begin{array}{r} 32 \\ \hline 8 \end{array}$$

$$\begin{array}{r} 2944 \\ \hline 736 \\ \hline 5 \\ \hline 8)3680(\end{array}$$

Answer, 460 Men.

The numbers of the above-stating are abbreviated, by first cutting off the Cyphers from the first and second numbers, and then dividing the first and third numbers by 6 and 4.

The reasons of this abbreviation may be seen in the fifth sort of Reduction of vulgar Fractions.

More Questions in the DIRECT RULE of THREE.

The cloathing of a Regiment of 740 men, comes to 3000 *l.* how much is that for each man?

*Answer, 4 *l.* 1 *s.* 0 *d.* $\frac{1}{2}$*

How long shall I be laying up 10000 *l.* if I put by $\frac{1}{2}$ a Guinea a week?

Answer, 366 years, 15 weeks, 4 days.

500 Seamen are to have 4 *d.* $\frac{1}{2}$ per day each, what will pay them for 23 months?

*Answer, 6037 *l.* 10 *s.**

What will an Estate of 4000 *l.* per Annum, allow a Gentleman to spend a day?

*Answer, 10 *l.* 19 *s.* 2 *d.**

A Gentleman's daily expences are 13 *d.* above which, he yearly lays up 600 Nobles, at 6 *s.* 8 *d.* each,

each, what is his Estate worth *per Annum*?

Answer, 447 *l.* 17 *s.* 11 *d.*

A man owing 736 *l.* 10 *s.* compounds with his Creditors for 7 *s.* 9 *d.* *per l.* what will that amount to?

Answer, 285 *l.* 7 *s.* 10 *d.* $\frac{1}{2}$.

If, at 5 *s.* *per ell*, I gain 8 *l.* *per Cent.* by my Cloth, what shall I gain *per Cent.* if I sell the ell at 6 *s.* 3 *d.*?

Answer, 35 *l.*

If Tobacco, which cost 13 *d.* *per lb.* be sold for 18 *d.* *per lb.* what is gained *per Cent*?

Answer, 38 *l.* 9 *s.* 2 *d.* $\frac{3}{4}$.

Bought 5 pieces of Holland, each containing 56 ells *Flemish*, at 3 *s.* 2 *d.* *per ell*, what shall I gain in the whole, if I sell it for 5 *s.* 8 *d.* *per ell English*?

Answer, 3 *l.* 5 *s.* 4 *d.*

I have by me 96 *lb.* of Cloves, which cost me 58 *l.* but some damage having happened to them, I am willing to lose 8 *l.* in the whole; at what rate must I sell them *per ounce*?

Answer, 7 *d.* $\frac{3}{4}$.

A Merchant sends over to France 482 tons of Lead, at 4 *l.* 10 *s.* *per Fodder*, i. e. 19 $\frac{1}{2}$ C. what quantity of Wine, at 30 *l.* *per pipe*, may he expect in return?

Answer, 74 pipes, 19 gallons.

A Merchant sends to Spain 1300 pieces of Broad Cloth, each piece 47 yards, at 15 *s.* 6 *d.* *per yard*, to have returns from thence; the one half in Wine, at 65 *l.* *per tun*; and the other half in Oranges, at 3 *l.* 10 *s.* *per Chest*; what quantity of each will he have?

Answer, { 364 Tuns, 1 Hoghead of Wine.
6764 Chests of Oranges.

Two men part, at the same time, from the same place; the one travels north 32 miles a day, the other 36 miles a day south; how long will it be before they are 2000 miles asunder, supposing them to travel 12 hours each day?

Answer, 29 days, 4 hours, 56 $\frac{1}{2}$ minutes.

The Indirect Rule of THREE.

IN the *Indirect Rule of Three*, the numbers are in reciprocal proportion; that is, the fourth number to be found, is to bear the same ratio to the second as the third does to the first, but in an inverted order; that is, the greater the third term is in respect to the first, the less must the fourth be in respect to the second.

This rule differs in its operation, from the Direct, in that, after the question is stated, and the numbers of the statings prepared (as in the Direct Rule) your first and second must be multiplied together, and your third number be your Divisor. The Quotient, as before, will be the answer.

E X A M P L E S.

(*Ex. 1st.*) What number of men must be employed, to finish, in 12 days, what 43 men would be 35 days about?

<i>days.</i>	<i>men.</i>	<i>days.</i>
If 35	43	12
43		
—		
105		
140		
—		
12)1505(5		

Answer, 125 Men.

(*Ex. 2^d.*)

(*Ex. 2d.*) How many yards of Stuff 3 *qrs.* wide, will hang a Room which requires 420 yards of 5 *qrs.* wide?

qrs. *yards.* *qrs.*
If 5 ————— 420 ————— 3

5

3)2100{

Answer, 700

The reason of this operation will appear plain (after what has been said in the Direct Rule) by considering the last example. Now it is clear, that if the Stuff, being 5 quarters wide, there are 420 yards required, then were the Stuff but 1 quarter wide, 5 times 420 yards, *viz.* 2100 yards must be allowed; consequently, if the Stuff be 3 quarters wide, one third part of those yards will be sufficient; therefore 2100 divided by 3, will give the true answer required, *viz.* 700 yards.

To know whether a Question belongs to the Direct or Indirect Rule of Three.

Observe, If the third number, being more than the first number, requires more, or, being less, requires less, 'tis *Direct*; but if the third number, being more, requires less, or being less, requires more, 'tis *Indirect*.

Or, without any regard to the distinction of *Direct* and *Indirect*; if *more* is required, let the lesser of the two extremes be the Divisor; if *less*, the greater.

More Questions in the Indirect Rule of Three.

If I lend *A.* 136*l.* for 3 months, how long must I keep 42*l.* of his, to requite myself?

Answer, 9 months, 2 weeks, 6 days.

If 46 Clerks in 32 days finish a piece of writing, in what time would 55 Clerks accomplish the same?

Answer, 26 days, 9 hours, 9 minutes.

A Garrison, consisting of 1539 men, being besieged, have provisions only for 12 days; but it being necessary they should hold out 3 weeks, how many men must be sent out?

Answer, 660 men.

The DOUBLE RULE of THREE.

Questions in this Rule have five numbers proposed, and are frequently answered by two statings, tho' they may be performed by one, as shall be shewn hereafter.

EXAMPLES.

EXAMPLES.

1 (Ex 1st.) The carriage of 32 hundred weight 56 miles, comes to 12 s. After the same rate, what must I pay to have 78 hundred weight carried 94 miles?

First, $\begin{array}{r} C. \\ 32 \end{array} \begin{array}{r} 3. \\ 12 \end{array} \begin{array}{r} C. \\ 78 \end{array}$

— s. d.
32)936(29 3facit.
64

	<i>Miles.</i>	<i>s.</i>	<i>d.</i>	<i>Miles.</i>	• 8
Then, If	56	— 29	3 —	94	12
		12			—

351

94

1404

3159

56)32994(589
280

499

448

• 514

504

30

12)589(1

210496

Answer, 2 l. 9 s. 1 d.

Note,

[37]

Note, The solution had been the same, if the miles had been made first and third numbers of the first stating; and the C. weights the first and third numbers of the last.

Note also, this example may be done by one stating, thus :

$$\begin{array}{rcl} \text{C.} & & \text{C.} \\ \text{If } 32 & \text{---} & 12 & \text{---} & 78 \\ & 56 \text{ Miles.} & & & 94 \text{ Miles.} \end{array}$$

$$\begin{array}{r} 192 \\ 160 \\ \hline 1792 \end{array}$$

$$\begin{array}{r} 312 \\ 702 \\ \hline 7332 \\ 12 \end{array}$$

$$\begin{array}{r} \text{---} \text{ s. d.} \\ 1792)87984(49 \text{ } 1 \\ 7168 \text{ or} \\ \text{---} \text{ l. s. d.} \\ 16304 \text{ } 2 \text{ } 9 \text{ } 1 \\ 16128 \end{array}$$

$$\begin{array}{r} \text{---} \\ \text{.. } 176 \\ 12 \end{array}$$

$$\begin{array}{r} 1792)2112(1 \\ 1792 \\ \hline 320 \end{array}$$

(Ex. 2d.) How many men must be employ'd to reap 420 Acres in 17 days, if there were required 37 men to reap 54 Acres in 5 days?

	<i>Acres.</i>	<i>Men.</i>	<i>Acres.</i>
First, If	54	37	420
			37
			2940
			1260
	<i>Days.</i>	<i>Men.</i>	<i>Days.</i>
Then, If	5	287	17
		5	
			17)1435(84
			136
			75
			68
			42
Ans ^w . 84 Men.		7	

Note, If you would work such questions of the *Double Rule of Three*, as have one of their proportions indirect, by one stating; you must multiply the third number of your stating, by that number you would otherwise have placed under your first; and your first number by that you would have placed under the third; as in the following Example.

EXAMPLE.

EXAMPLE.

Acres.	Men.	Acres.	
If 54	37	420	} The number of days which have relation to the 54 Acres.
* 17		5	

378	2100
54	37
918	14700
	6300

* The number of days which have relation to the 420 Acres.

	Men.
918) 77700	(84 answer as before.
	7344
	4260
	3672
	588

Dr. Harris, in his *Lexicon Technicum*, teaches another way of ranging the five Numbers given in a *Double Rule of Three Question*; which, according to the following Rules, gives the solution, whether the proportion be direct or indirect. Take his method in his own words:

I. "Observing, that the given terms are always five, whereof three are conditional and antecedent, or supposititious, the other two demand the question, and are consequents answering some of the former antecedents; insomuch, that with the answer, there will be as many consequents as antecedents, which must match one another in the same denomination exactly.

II. "For the right placing of the question and terms, the three terms of the conditional part are duly to be regarded: Let that which is the principal cause of Loss or Gain, Increase or Decrease, Action or Passion, be put in the first Place; and that

“ which betokeneth the space of time, distance of
 “ place, &c. be put in the second place ; and the re-
 “ maining part in the third. The conditional part
 “ thus stated, the other two terms, wherein the de-
 “ mand lies, must be placed so under the former
 “ terms, that they may correspond one with another.

R U L E 1.

“ Then, if the blank, or place sought, fall under the
 “ third term, multiply the three last terms for a Divi-
 “ dend, and the two first for a Divisor ; and the Quo-
 “ tient gives the sixth term required.

R U L E 2.

“ But if the blank fall under the first or second
 “ term, multiply the 1st, 2d, and 5th terms for a Di-
 “ vidend, and the 3d and 4th for a Divisor : The
 “ Quotient gives the Answer.

E X A M P L E 1.

“ If 12 Rods of ditching be done by 2 men in 6
 “ days, how many Rods shall be wrought by 8 men
 “ in 24 days ?

Answer, 192.

The numbers being stated as before directed, they
 will stand thus, the blank falling under the third place.

<i>Men.</i>	<i>Days.</i>	<i>Rods.</i>
2	6	12
8	24	

Therefore, by the first Rule, the three last terms,
viz. 12, 8, and 24, being multiply'd into each other,
 give 2304 for a Dividend ; and the two first terms,
viz. 2 and 6, multiply'd together, give 12 for a Di-
 visor, which quotes 192, the answer.

E X A M P L E.

EXAMPLE 2.

If 2 men work 12 Rods in 6 days, how many men will work 192 Rods in 24 days?

Men.	Days.	Rods.
2	6	12
	24	192

Here, according to the second Rule, the first, second, and fifth terms multiply'd, give 2304, which divided by 288, the product of the third and fourth numbers, quotes 8 the answer.

More EXAMPLES.

If 400 pecks of Corn will serve 32 horses 108 days, what quantity will 500 horses eat in 20 days?

Answer, $1157\frac{2}{3}$ pecks.

If 56 gallons of Drink serve 25 persons 120 days, how long will 200 gallons serve 12 persons?

Answer, $892\frac{5}{7}$ days.

If a certain quantity of provision serves 30 men for 50 days with 24 ounces each, how much must 90 men have apiece when they are oblig'd to make the same provision serve them 60 days?

Ounces. Drams.

Answer, 6 ——— $10\frac{2}{3}$.

Mr. Gardiner has publish'd, in the explication of his Tables of Logarithms, a peculiar method of resolving the several kinds of this Rule, (taken from the Papers of *W. Jones*, Esq; F. R. S.) which is not only excellent in the use of Logarithms, but also in the way of common Arithmetic.

1. "Set down the terms expressing the condition of the question in one line, and in any order.

C 3

2. "Under

2. " Under each conditional term set its corresponding one, in another line.

3. " Multiply the producing terms of one line, and the produced term of the other line, continually, and take the result for a dividend.

4. " Multiply the remaining terms continually, and let their product be the divisor.

5. " The quotient of this division will be the term required.

" By producing terms here is meant, whatsoever necessarily and jointly produce any effect: As the cause, and the time; length, breadth and depth; buyer and his money; seller and his goods; all necessarily inseparable in producing their several effects."

In a question where a term is understood, and not expressed, (as in the third Example) that term may be represented by Unity.

This Rule is so general, as to comprehend all cases that come under the common Rule of Three, whether direct, or inverse; whether single, or any how compounded; so that questions in any of the following Rules, where proportion is used, may be readily solved thereby.

EXAMPLES.

Ex. 1. Take the same as in page 38. Putting down the terms expressing the condition in the first line, and under each term its corresponding one in the second, filling the blank space (or term sought) with a capital Q thus:

<i>Men.</i>		<i>Days.</i>		<i>Acres.</i>
If 37	require	5	to reap	54
Q		17		420

Here

Here it is plain, that the produced terms are the acres, and the men and days are the terms producing the quantity of acres reaped ; therefore looking in page 128 for the characters here us'd, and particularly the latter part of that for division, it will be that

$$Q = \frac{420 + 37 + 5}{54 + 17} = \frac{70 + 37 + 5}{9 + 17} = \frac{12950}{153} = 84 \text{ Men.}$$

Ex. 2. Take that in page 41, thus :

<i>Rods.</i>	<i>Men.</i>	<i>Days.</i>
If 12	2	6
Q	8	24

Here the produced terms are the rods, and the producing terms the men and days, as before.

$$Q = \frac{12 + 8 + 24}{2 + 6} = 8 + 24 = 192 \text{ Rods.}$$

Note, When any of the terms in the dividend are the same as in the divisor, they may be struck out in both ; or when any in both may be equally divided, the quotients may be put instead of those terms, as in the first example.

Ex. 3. If 54 men can build a Fort in 18 days, when working 17 hours in each day ; in how many days will 120 men build the same, in working but 12 hours each day ?

<i>Men.</i>	<i>Days.</i>	<i>Hours.</i>	<i>Fort.</i>
If 54	18	17	1
120	Q	12	1

Here the Fort being 1 is the term produced, and the men, days and hours, the producing terms ; therefore

$$Q = \frac{54 + 18 + 17}{120 + 12} = \frac{27 + 17}{40} = \frac{459}{40} = \begin{matrix} \text{Ds. Hrs. Min.} \\ 11 : 5 : 42 \end{matrix}$$

Ex. 4. What is the Interest of 572 l. for 8 months, at 5 l. per Cent. per Annum ? See page 68.

If 100 *l.* ——— 5 *l.* ——— 12 Months
 572 *Q* 8

Here the Interest is the term produced, and the principal and months the producing terms; therefore

$$Q = \frac{572 + 8 + 5}{100 + 12} = \frac{572}{30} = 19\frac{2}{3} = 19 \text{ l. } 1 \text{ s. } 4 \text{ d.}$$

Ex. 5. If 250 men in 5 days, working 16 hours each day, will dig a trench 240 yards long, 5 yards wide, and 4 deep; in how many days, working but 10 hours in each day, will 24 men dig a trench 360 yards long, 4 wide, and 3 deep?

<i>Men.</i>	<i>Days.</i>	<i>Hours.</i>	<i>long.</i>	<i>wide.</i>	<i>deep.</i>
If 250 ———	5 ———	16 ———	240 ———	5 ———	4
24 <i>Q</i>	10	360	4	3	

Here the produced terms are the trenches, consisting of length, width and depth, and the men, days and hours, the producing terms; therefore

$$Q = \frac{360 + 4 + 3 + 250 + 5 + 16}{240 + 5 + 4 + 24 + 10} = 75 \text{ days.}$$

As the Tables of Logarithms, when truly printed, are of excellent use to the proficients in Arithmetic; and having above an occasion given for mentioning those published by Mr. *Gardiner*, I think my Readers should be inform'd of their being the most correct, as well as complete, of any extant; and that he has publicly advertiz'd ten shillings reward to be shewn one error, even of a logarithm not within the half unit of the lowest place, or of a difference untrue between them, tho' given to every ten seconds of the Quadrant.

His Explication contains many curious solutions in logarithms, and very easy examples of getting the logarithm of any number, and the number to any logarithm, true to 19 places of figures, by tables therein given to 20 places; besides those of all numbers from 1 to 102100, with parts for two figures farther, every where in the same openings as the logarithms of the first five or six figures.

C H A P. IV.

Of PRACTICE, and TRET and TARE.

BY PRACTICE, Merchants compendiously cast up the Price of their Commodities.

And by TRET and TARE, deduct their Allowances.

Both these Rules will be best explain'd by Examples; and both require the perfect Knowledge of the following

T A B L E.

Of a Pound.		Of a Shilling.		Of a Ton.	
s.	d.	d.		C.	
The even Parts	1	0	$\frac{1}{20}$	2	$\frac{1}{10}$
	1	8	$\frac{1}{12}$	2 $\frac{1}{2}$	$\frac{1}{8}$
	2	0	$\frac{1}{10}$	4	$\frac{1}{5}$
	2	6	$\frac{1}{8}$	5	$\frac{1}{4}$
	3	4	$\frac{1}{6}$	10	$\frac{1}{2}$
	4	0	$\frac{1}{5}$	Of a Hund.	
	5	0	$\frac{1}{4}$	lb.	
	6	8	$\frac{1}{3}$	14	$\frac{1}{8}$
	10	0	$\frac{1}{2}$	16	$\frac{1}{7}$
				Of $\frac{1}{2}$ C.	
		Of a Pen.		lb.	
		Far.		7	$\frac{1}{8}$
		1	$\frac{1}{4}$	8	$\frac{1}{7}$
		2	$\frac{1}{2}$	14	$\frac{1}{4}$

Before we proceed to the rules of Practice, it will be proper to shew the manner of multiplying and dividing numbers of several denominations, by a single figure, without Reduction.

Example of MULTIPLICATION.

	<i>l.</i>	<i>s.</i>	<i>d.</i>	
<i>Multiply</i>	5	8	4	<i>by 8.</i>
			8	
<i>Product</i>	43	6	8	

Here, first, the 4 *d.* multiply'd by 8, gives 32 *d.* equal to 2 *s.* 8 *d.* therefore setting down 8 in the place of pence, we carry 2 to the produce of the shillings. Again, 8 times 8 *s.* being 64 *s.* and the 2 carry'd making it 66 *s.* or 3 *l.* 6 *s.* the 6 is set down in the place of shillings, and the 3 carry'd on to the Produce of the Pounds. Lastly, 8 times 5 *l.* being 40 *l.* and the 3 carry'd making 43 *l.* the whole product appears to be 43 *l.* 6 *s.* 8 *d.*

Example of DIVISION.

	<i>l.</i>	<i>s.</i>	<i>d.</i>	
<i>Divide</i>	43	6	8	<i>by 8</i>
	8)	43	6	8 (
<i>Quotient</i>	5	8	4	

In dividing the above sum, we find, first, that 8 will go 5 times in 43 *l.* and that 3 will remain; therefore setting down 5 in the place of pounds, we multiply the said remainder by 20, and taking in the 6 odd shillings, the produce is 66 *s.* in which again, the divisor going 8 times, and 2 remaining, we set down 8 in the place of shillings, and multiply the 2 remaining by 12, which, with the 8 odd pence

pence taken in, produces $32d.$ in which, lastly, the Divisor 8 going exactly 4 times, 4 is set in the place of pence, and the whole Quotient will be found to be $5\text{ l. } 8\text{ s. } 4\text{ d.}$

Rules of PRACTICE, with Examples.

1. When the Price is such a number of pence as make an even part of a shilling, divide the quantity of goods by such part, and the quotient will be the answer in shillings.

E X A M P L E.

Suppose you would know the price of 5296 ounces of Gum, at $6d.$ per ounce; then

$$\begin{array}{r} 6d. \text{ being } \frac{1}{2} \} 5296 \\ \text{of a Shilling, } \} \hline 2648 \text{ shillings is the Answer.} \end{array}$$

For were the price $12d.$ or $1s.$ the number of ounces would be the number of shillings they would cost: Therefore the price being $6d.$ the $\frac{1}{2}$ of a shilling, the answer must be but $\frac{1}{2}$ so many shillings. So at $1d.$ $1\frac{1}{2}d.$ $2d.$ $3d.$ or $4d.$ 'tis but the $\frac{1}{12}$, $\frac{1}{8}$, $\frac{1}{6}$, $\frac{1}{4}$, or $\frac{1}{3}$ parts of the quantity of goods, and the quotient (as aforesaid) will be the answer in shillings; which divided by 20, gives the pounds.

Note, If any thing remains in dividing by the said parts (pence being the next denomination) it must be multiply'd by 12, the pence in 1 shilling; and the product divided again by the said part; and if any thing remains there, it must be multiply'd by 4, and still divided by the said part, to produce farthings.

E X A M P L E.

$$\begin{array}{r}
 \text{oz.} \quad d. \\
 5243 \text{ at } 1 \frac{1}{2} \text{ per Ounce.} \\
 d. \quad 1 \frac{1}{2} \frac{1}{8} \left| \begin{array}{r} 5243 \\ \hline 6554 \frac{1}{2} \end{array} \right.
 \end{array}$$

Answer, - - 32 l. 15 4d. $\frac{1}{2}$

2. If the pence of the price are not an even part of a shilling, it will require more than one division, to find the cost of the goods. Thus at 5d. or any other number of pence between 6 and 12.

E X A M P L E S.

$ \begin{array}{r} \text{lb.} \quad d. \\ 527 \text{ at } 5 \text{ per lb.} \\ d. \quad 4 \frac{1}{3} \left \begin{array}{r} 175 \quad 8 \\ 43 \quad 11 \\ \hline 219 \quad 7 \end{array} \right. \end{array} $	$ \begin{array}{r} \text{lb.} \quad d. \\ 436 \text{ at } 9 \text{ per lb.} \\ d. \quad 6 \frac{1}{2} \left \begin{array}{r} 218 \\ 109 \\ \hline 327 \end{array} \right. \end{array} $
<i>Ans.</i> 10 l. 19 s. 7 d.	<i>Ans.</i> 16 l. 7 s.

In the first of these, 5 d. is taken, thus : 4 d. is the $\frac{1}{3}$ of a shilling, or the top line, and 1 d. the $\frac{1}{4}$ of 4 pence.

In the second, the $\frac{1}{2}$ of a shilling is first taken for 6d. and then the $\frac{1}{2}$ of that 6 d. for 3 d.

3. If the price is both pence and farthings ; for the pence, as before ; and for the farthings, take the even parts of a penny : thus,

E X A M P L E.

E X A M P L E.

	oz.	d.
	532	at $1 \frac{1}{4}$ per Ounce.
1 Penny, $\frac{1}{2}$	44	4
1 Farthing, $\frac{1}{4}$	11	1
	515	5
Answer,	2l. 15s.	5d.

4. When the price is more than 1s. and less than 2s. leave the top line for the shilling, and take your parts, as before, for the remaining pence.

E X A M P L E.

d.	lb.	d.
	5437	at $19 \frac{1}{2}$ per lb.
6	2718	6
$1 \frac{1}{2}$	679	7 $\frac{1}{2}$
	88315	1 $\frac{1}{2}$
Answer.	441l.	15s. 1 $\frac{1}{2}$ d.

5. If the price is two or more shillings, with pence, &c. either take the even parts of a pound from your top line for the shillings, which will give you the price in pounds; or multiply your top line by the number of shillings, which will give the answer in shillings, still working for the pence as before. See both ways in the example.

E X A M P L E.

E X A M P L E.

s. d.	oz. s. d. 5263 at 2 6 per Oz.	oz. s. d. 5263 at 2 6 per Oz.
2 6 $\frac{1}{8}$	657 17 6	$d. \left \frac{1}{2} \right$ 6 $\frac{1}{2}$ 10526 2631 6 1315 7 6

Answer, 657*l.* 17*s.* 6*d.*

By the first way, it is thus done: Had the price been 1*l.* or 20*s.* 'tis plain it would have come to as many pounds as there were ounces; therefore at 2*s.* 6*d.* which is the $\frac{1}{8}$ of a pound, it must come to the $\frac{1}{8}$ of so much money.

The other is done by supposing the top line at 1*s.* then for 2*s.* it must be twice so much; and for the 6*d.* half the top line, or the value of a shilling, must be added. See both ways in another.

E X A M P L E.

s.	lb. s. d. 5276 at 19 9 per lb.	lb. s. d. 5276 at 19 9 per lb.
10 $\frac{1}{2}$	2638	$d. \left \frac{1}{2} \right$ 5276 19 47484 5276 6 $\frac{1}{2}$ 2638 3 $\frac{1}{2}$ 1319 10420 1
5 $\frac{1}{2}$	1319	
4 $\frac{1}{2}$	1055 4	
6 <i>d.</i> $\frac{1}{8}$	131 18	
3 $\frac{1}{2}$	65 19	

Answer, 5210*l.* 1*s.*

Answer, 5210*l.* 1*s.*

6. When

6. When the Price is any thing between one and two pounds, leave the top line for the pound, and take your parts for the remainder.

E X A M P L E.

	lb.	s.	d.	
s. d.	5384	at	22 6	per lb.
2 6 $\frac{1}{8}$	673			

Answer, 6057*l.*

7. When the price is two or more pounds, multiply the top line by the number of pounds; and for the remainder take parts, as before.

E X A M P L E.

lb.	l.	s.	d.
2364	at	4 18	10 $\frac{1}{2}$ per lb.
4			

3.		9456		
10	$\frac{1}{2}$	1182		
5	$\frac{1}{2}$	591		
2	$\frac{1}{3}$	236	8	
1	$\frac{1}{2}$	118	4	
6d.	$\frac{1}{2}$	59	2	
3	$\frac{1}{2}$	29	11	
1 $\frac{1}{2}$	$\frac{1}{2}$	14	15	6

Answer, 11687*l.* 00*s.* 6*d.*

8. When with the given quantity there are odd parts, such parts must be taken from the given price; as in the following examples.

E X A M P L E S.

EXAMPLES.

	C.	qrs.	l.	s.	
	365	2	at	5	5 per C.
	5				
s.	1825				
5	$\frac{1}{4}$	91	5		
2 qrs. $\frac{1}{2}$		2	12	6	

Answer, 1918l. 17l. 6d.

	C.	qrs.	lb.	l.	s.	d.
	7365	3	14	at	8	19 6 per C.
	8					
s.	58920					
10 . . $\frac{1}{2}$	3682	10				
5 . . $\frac{1}{2}$	1841	5				
4 . . $\frac{1}{8}$	1473	0				
6d. . . $\frac{1}{8}$	184	2	6			
2 qrs. $\frac{1}{2}$	4	9	9			
1 . . $\frac{1}{2}$	2	4	10 $\frac{1}{2}$			
14 lb. $\frac{1}{2}$	1	2	5 $\frac{1}{4}$			

Answer, 66108l. 14s. 6d. $\frac{3}{4}$

As in the first of these, for the 2 qrs. $\frac{1}{2}$ the price of the C. is taken; so in the second, not only $\frac{1}{2}$ the price of the C. is taken for 2 qrs. but also $\frac{1}{2}$ the price of 2 qrs. for 1 qr. and $\frac{1}{2}$ that again for 14 lb. See more Examples.

EXAMPLES.

EXAMPLES.

Tons. C. grs. lb. l. s. d.
573 13 2 18 at 9 12 10 per Ton.
9

5157
10s. . . $\frac{1}{2}$. 286 10
2 . . . $\frac{1}{5}$. 57 06
10d. . $\frac{1}{12}$. 23 17 6
10 C. $\frac{1}{2}$. . 4 16 5
2 . . . $\frac{1}{5}$. . . 19 $3\frac{1}{4}$
1 . . . $\frac{1}{2}$. . . 09 $7\frac{1}{2}$
2 grs. $\frac{1}{2}$. . . 04 $9\frac{3}{4}$
14 lb. $\frac{1}{4}$. . . 01 $2\frac{1}{4}$
2 . . . $\frac{1}{7}$ 2
2 . . . $\frac{1}{7}$ 2

Answer, 5531 l. 5 s. 1 d. $\frac{3}{4}$

lb. oz. dr. l. s. d.
527 13 15 at 2 8 7 $\frac{1}{2}$ per lb.
2

1054
5s. . . $\frac{1}{4}$. . 131 15
2 . . . $\frac{1}{10}$. . 52 14
1 . . . $\frac{1}{2}$. . 26 07
6d. . . $\frac{1}{2}$. . 13 03 06
1 $\frac{1}{2}$. . . $\frac{1}{4}$. . 3 05 $10\frac{1}{2}$
8 oz. . . $\frac{1}{2}$. . 1 04 $03\frac{3}{4}$
4 . . . $\frac{1}{2}$. . 12 01 $\frac{3}{4}$
1 . . . $\frac{1}{4}$. . 03 00 $\frac{1}{4}$
8 drs. $\frac{1}{2}$. . 01 06
4 . . . $\frac{1}{2}$. . 09
2 . . . $\frac{1}{2}$. . 04 $\frac{1}{2}$
1 . . . $\frac{1}{2}$. . 02 $\frac{1}{4}$

Answer, 1283 l. 7 s. 8 d.

The

[54]

The last Sum another way.

lb. oz. drachms. l. s. d.

527 13 15 at 2 8 7 $\frac{1}{2}$ per lb.

48

4216

2108

6 d.	$\frac{1}{2}$	..	263	6
1 $\frac{1}{2}$	$\frac{1}{4}$	...	65	$10\frac{1}{2}$
8 oz.	$\frac{1}{2}$	...	24	$3\frac{1}{4}$
4 ..	$\frac{1}{2}$	...	12	$1\frac{1}{4}$
1 ..	$\frac{1}{4}$	...	3	$0\frac{1}{4}$
8 dr.	$\frac{1}{2}$	...	1	6-
4 ..	$\frac{1}{2}$	...	...	9-
2 ..	$\frac{1}{2}$	...	...	$4\frac{1}{2}$
1 ..	$\frac{1}{2}$	...	...	$2\frac{1}{2}$

2566 $\frac{1}{7}$ 8

Ans. 1283 l. 7s. 8d.

Another EXAMPLE.

lb. Troy. oz. dwts. grs. l. s. d.

736 11 12 13 at 23 16 4 per lb.

23

2208

1472

10 s.	$\frac{1}{2}$	..	368	
5 ..	$\frac{1}{2}$	..	184	
1 ..	$\frac{1}{3}$	..	36	16
4 d.	$\frac{1}{3}$	..	12	5
6 oz.	$\frac{1}{2}$	..	11	18
4 ..	$\frac{1}{3}$	..	7	18
1 ..	$\frac{1}{4}$	..	1	19
10 dwts.	$\frac{1}{2}$	..	19	10
2 ..	$\frac{1}{5}$	..	3	$11\frac{1}{2}$
12 grs.	$\frac{1}{4}$	..	..	$11\frac{1}{4}$
1 ..	$\frac{1}{12}$	..	..	$\frac{3}{4}$

Ans. 1755 l. 2s. 9d. $\frac{1}{2}$

Note,

Note, The foregoing examples, and all other questions in *Practice*, may be performed by the *Rule of Three*; *Practice* being only a compendious way, of calculating such propositions in proportion, as have an unit for their first number: But to shew the brevity of this way of working, as well as to prove the truth thereof; see the last example done by the *Rule of three*, as follows.

By the *Rule of Three* thus:

lb.	l.	s.	d.	lb.	oz.	dwt.	grs.
If 1—cost—	23	16	4	what will	736	11	12 13?
12	20			12			
12	476			8843			
20	12			20			
240	5716			176872			
24				24			
960				707491			
480				353745			
5760				4244941			
				5716			
				25469646			
				4244941			
				29714587			
				21224705	12		
				57610)242640827516(	4212514(	10	
				1224	210)3510412		
				720	<i>Answer, 17552 2 10½</i>		
				1448			
				2962			
				827			
				2515			
				2116			
				4			
				57610)84614(			
				270			

Three Farthings more than by Practice lost there by the Fractions remaining.

PROOF.

P R O O F.

Sums in this rule may be proved either by taking the parts two ways, as the last example but one ; or by the *Rule of three*, as the last.

A B B R E V I A T I O N S.

As this whole rule is indeed nothing else, it may be thought strange to mention short ways here, in a particular section ; but as there are degrees of comparison, so even short ways may be more shorten'd.

1. The value of any quantity (the given price of one of which is an even number of shillings) may be most compendiously found, by multiplying the said quantity by $\frac{1}{2}$ their number, doubling the Units of the first product for shillings, the rest being pounds.

E X A M P L E.

5276 Ells of Holland, at 8 s. per Ell.

4

Product, 2110: 8

2. If the price is an odd number of shillings, it is but doing as before, and adding $\frac{1}{20}$ of the top line to the product.

E X A M P L E.

Yards. s.

5278 at 15 per Yard.

7

3694: 12

1 s. $\frac{1}{20}$. 263: 18

3958 l. 10 s. Answer.

3. Were

3. Were there pence in the price, their parts might also easily be taken in the same method.

E X A M P L E.

	lb.		s.	d.
	5384	at	7	7 $\frac{1}{2}$ per lb.
	3			
	1615		4	
1s. $\cdot \frac{1}{20}$	269		4	
6d. $\cdot \frac{1}{2}$	134		12	
1 $\frac{1}{2}$ $\cdot \frac{1}{4}$	33		13	

Answer. 2052l. 13s.

4. Another short and useful way, when the quantity is small, is to multiply the price by it; thus, if your quantity is expressed by one figure, 'tis done by one line.

E X A M P L E.

	l.	s.	d.
5 Yards of Velvet,	at	1	13 6 per Yard.
		5	

Product. 8l. 7s. 6d.

5. So any quantity that can be expressed by 2 figures, and some of 3, may be easily and speedily done thus: Find out what two figures, multiplied together, will produce the given quantity, (as suppose 32 yards or pounds, &c. the given quantity which is produced by the multiplication of 8 by 4) and multiply the price by one of the said figures; and the product of that, multiplied again by the other, will give the answer.

E X A M P L E S.

EXAMPLES.

Yards. s. d.
32 at 15 8½ per Yard.
8

The value of 8 yards, 6 5 8—
which multiplied by 4

gives 25 l. 2 s. 8 d. the price of 32 yards,
—4 times 8 being 32.

Yards. l. s. d.
144 at 1 5 6 per Yard.
12
15 6 0
12

Product 183 l. 12 s. 0 d.

6. If no two figures will exactly produce those given, take such as will give the nearest product to them, which will generally be within one or two, more or less; the value of which must accordingly be added or subtracted.

EXAMPLE.

lb. l. s. d.
55 at 1 10 9 per lb.
9

The price of 9 lb. 13 16 9
which multiplied by 6

gives the price of 54 lb. 83 00 6
to which the price of 1 lb. added 1 10 9

lb.
makes 84 11 3 the value of 55.
So,

So, had it been multiplied by 7 and 8, which would have given the value of 56 lb. too much by 1 lb. the price of 1 lb. must have been subtracted.

7. If the quantity has odd parts, it will be easy taking the value of such parts from the given price; as in the following

EXAMPLE.

<i>Yds.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>	
43 $\frac{1}{2}$ at	2	12	7	<i>per Yard.</i>
			7	
	18	8	1	
			6	
	110	8	6	
<i>qrs.</i>		2	12	7
2 $\frac{1}{2}$		1	6	3 $\frac{1}{2}$
<i>Product.</i>	114	l.	7	s. 4 $\frac{1}{2}$

TRETT and TARE.

TARE and **TRETT** are allowances made to Merchants in buying their goods.

Tare, of what they can agree for *per* the whole, *per* chest, &c. or *per* C. for the weight of the bag, box, chest, &c. which contains the commodity.

Trett, for the waste, moles, or dust; and is always 4 lb. *per* 104 lb.

There is also another allowance sometimes given of 2 lb. for every 3 C. for the turn of the scale, called cloff, or clough.

Note, the whole weight, before any allowances are made, is called *gross*; when part is deducted, the remainder is called *futtle*; but when all are taken from it, what's left, is called *neat*.

RULES,

RULES, with EXAMPLES.

1. The *Tare* of any quantity of goods may easily be found, if at so much in the whole, only by subtracting the said allowance from the gross weight; if at so much *per chest*, &c. by multiplying the pounds tare by the number of chests, &c. and subtracting as before; and if at so much *per C.* by taking such part or parts of the gross weight, as the allowance is of a C.

EXAMPLES.

First sort. Suppose 15 C. 2 qrs. 13 lb. tare were allowed on 456 C. 1 qr. 19 lb. of tobacco, what would be the neat weight?

	C.	qrs.	lb.	
From	456	1	19	Gross.
Subtract	15	2	13	Tare.
Remainder	440	3	6	Neat.

Second sort. What's the neat weight of 3 frails of raisins, each weighing 3 C. 2 qrs. 10 lb. gross; tare at 20 lb. *per frail*?

C.	qrs.	lb.	lb.
3	2	10	20
3 Frails.			3 Frails.
10	3	2	Gross.
	2	4	Tare.
			60 lb. or 2 qrs. 4 lb.

Answer, 10 0 26 *Neat.*

	C.	qrs.	lb.	lb.
<i>Third sort.</i>	246	3	12	Gross.
				Tare 14 per C.
lb.				
14 $\frac{1}{8}$	30	3	12	Tare.

Answer, 216 0 0 *Neat.*

Again,

	C.	qrs.	lb.		lb.
Again,	364	1	19	Grofs.	Tare 16 per C.
	lb.				
	16 $\frac{1}{7}$	52	0	6 $\frac{1}{2}$	Tare.

Answer, 312 1 12 $\frac{1}{2}$ Neat.

Note, 14 and 16 pounds may be called the standards of Tare; for from them may any other number of pounds, more or less, be taken, as in the following

E X A M P L E S.

	C.	qrs.	lb.		lb.
	573	2	13	Grofs.	Tare 17 per C.
	lb.				
	14 $\frac{1}{8}$	71	2	22 $\frac{1}{2}$	
	2 $\frac{1}{7}$	10	0	27-	
	1 $\frac{1}{2}$	5	0	13 $\frac{1}{2}$	
	87	0	7	Tare.	

Answer, 486 2 6 Neat.

	C.	qrs.	lb.		lb.
	545	3	11	Grofs.	Tare 10 per C.
	lb.				
	16 $\frac{1}{7}$	78	1	17 $\frac{1}{2}$	
	8 $\frac{1}{2}$	39	0	22 $\frac{3}{4}$	
	2 $\frac{1}{4}$	9	3	5 $\frac{1}{2}$	
	49	0	0 $\frac{1}{4}$	Tare.	

Answer, 499 3 10 $\frac{3}{4}$ Neat.

Note, 'Tis first found what Tare 16 lb. would produce; because 8 lb. being the $\frac{1}{4}$ part of a C. would be a troublesome divisor.

[62]

2. *Trett* being always 4 lb. per 104 lb. the constant method of finding it, is by taking the $\frac{1}{26}$ part of the line it is to be deducted from, 4 times 26 being 104. But it being difficult to divide by 26 in one line, it will be more safe and easy to do the work aside, as in the first Example.

EXAMPLES.

	C.	qrs.	lb.		lb.
	428	1	19	Gross.	<i>Trett</i> 4 per 104
lb.					
428	16	1	25 $\frac{1}{2}$	<i>Trett.</i>	26)428(16
<i>facit</i>	411	3	21 $\frac{1}{2}$	<i>Neat.</i>	168
					12
					4
					26)49(1
					23
					28
					193
					47
					26)663(25
					143
					13
					4
					26)52(2
					0

[63]

	C.	qrs.	lb.	lb.
	836	2	17	Gross, Tare 22 per C.
				Trett, 4 lb. per 104 lb.
lb.				
16 $\frac{1}{7}$	119	2	2 $\frac{1}{4}$	
4 $\frac{1}{4}$	29	3	14 $\frac{1}{2}$	
2 $\frac{1}{2}$	14	3	21 $\frac{1}{4}$	
	164	1	10	Tare.
	672	1	7	Suttle.
lb.				
4 $\frac{1}{26}$	25	3	12	Trett.
facit	646	1	23	Neat.

3. *Clough* (which is always 2 lb. for 3 C.) may be found by taking the $\frac{1}{108}$ part of the line it is to be deducted from, 2 lb. being the $\frac{1}{108}$ part of 336 lb. or 3 C. But let the work be done aside, as directed in finding the Trett. Or the allowance for *Clough* may be found, by first dividing the hundreds of the line it is to be taken from, by 3, which brings them into 3 C's; then 2 lb. being to be allowed for every 3 C. as many 3 C's as it produces, so many 2 lb's it will allow; then dividing by 56 (the double pounds in a hundred) the quotient will be hundreds, and the remainder double pounds; to which adding, what may be allowed for the odd hundreds, quarters, and pounds of the given weight, it makes the whole *Clough*; which subtracted from the said given weight, leaves the Neat. See both ways in the first Example.

EXAMPLE 1.

What will be the neat weight of 5647 C. 3 qrs. 13 lb. *Gross*, allowing for Clough 2 lb. for 3 C?

C.	qrs.	lb.
5647	3	13 <i>Gross</i> .
33	2	13 <i>Clough</i> .
5614	1	00 <i>Neat</i> .
C.		
168)5647(33		3)5647(1
•607		56)1882(33 2 12
103		202
4		
		14)34(2
qrs.		
168)415(2		•6
•79		
28		
635		
159		
lb.		
168)2225(13		
•545		
•41		

Note, The remainder 34 (which are double pounds) is divided by 14, the double pounds in a qr. to bring the said remainder into quarters of an hundred, making 2 qrs. and 12 lb. And 1 lb. being allowed for the C. 3 qrs. and 13 lb. the whole makes, as before, 33 2 qrs. 13 lb.

EXAMPLE 2.

What's the neat weight of 3 hogsheads of tobacco,
weighing, viz.

Number.	C.	qrs.	lb.
1	5	3	18
2	4	2	11
3	5	1	19

lb. 15 3 20 *Gross.* { *Tare 7 lb. per C.*
Trett 4 lb. per 104 lb.
Clough 2 lb. for 3 C.

14 $\frac{1}{8}$ 1 3 27 $\frac{1}{2}$

7 $\frac{1}{2}$ 3 27 $\frac{1}{2}$ *Tare.*

14 3 20 $\frac{1}{2}$ *Suttle.*
 4 $\frac{1}{8}$ 2 8 $\frac{1}{4}$ *Trett.*

14 1 12 $\frac{1}{4}$ *Suttle.* C.
 9 $\frac{1}{2}$ *Clough.* 3) 14(4

Ans. 14 1 2 $\frac{3}{4}$ *Neat.* 2 lb.

Note, The 4 three hundreds allow — 8

And the odd 2 C. 1 gr. 14 lb. $\frac{1}{2}$ allow — 1 $\frac{1}{2}$

Total Clough 9 $\frac{1}{2}$

CHAP. V.

Of INTEREST.

INTEREST is either { Simple, { which arises only from
 Or { the Principal.
 Compound, { which arises both from
 Principal and Interest.

SIMPLE INTEREST.

SIMPLE INTEREST being one of the branches of Proportion, or the *Rule of Three*, is worked as follows.

1. The Interest of any Sum for one year is found by this plain proportion, *viz.*

As 100 *l.*
 To the rate of Interest :
 So is the given Principal,
 To the Interest required.

EXAMPLE.

What's the Interest of 429 *l.* for a year, at 6 *l.* per Cent. per Annum?

$$\begin{array}{r} \text{l.} \qquad \qquad \text{l.} \qquad \qquad \text{l.} \\ \text{If } 100 \text{ --- } 6 \text{ --- } 429 \\ \qquad \qquad \qquad 6 \end{array}$$

Answer, 25*l.* 14*s.* 9*d.* $\frac{1}{2}$

$$\begin{array}{r} 27 \overline{) 74} \\ \underline{20} \\ 14 \overline{) 80} \\ \underline{12} \\ 9 \overline{) 60} \\ \underline{4} \\ 2 \overline{) 40} \end{array}$$

Note. The cutting off two figures to the right-hand, divides such lines by 100. See *Abbreviations of Division*.

2. To find the interest of any sum for 2 or more years, multiply the year's amount by the number of years required.

EXAMPLE.

What interest will the foregoing principal produce in 4 years, at the same rate of interest?

$$\begin{array}{r} \text{l.} \quad \text{s.} \quad \text{d.} \\ 25 \quad 14 \quad 9\frac{1}{2} \text{ the interest for one year.} \\ \underline{\qquad 4 \qquad} \end{array}$$

Product 102 19 2

3. To find interest for months, divide the year's amount by the even parts they make of a year.

E X A M P L E.

What's the interest of 572 *l.* for 8 months, at 5 *l.* per Cent. per Annum?

$$\begin{array}{r}
 \text{l.} \quad \text{l.} \quad \text{l.} \\
 \text{If } 100 \text{ --- } 5 \text{ --- } 572 \\
 \quad \quad \quad 5 \\
 \quad \quad \quad \hline
 \quad \quad 28160 \\
 \quad \quad \quad 20 \\
 \quad \quad \quad \hline
 \quad 12100
 \end{array}$$

$$\begin{array}{r}
 \text{l.} \quad \text{s.} \\
 28 \quad 12 \\
 \hline
 \text{M.} \\
 6 \frac{1}{2} \quad 14 \quad 6 \\
 2 \frac{1}{3} \quad 4 \quad 15 \quad 4 \\
 \hline
 \end{array}$$

*Answer, 19 *l.* 1 *s.* 4 *d.**

4. If the time is weeks or days, instead of taking the even parts of a year as for months, it must be done by a second stating in the *Rule of Three*.

E X A M P L E S.

Ex. 1. What's the interest of 321 *l.* 16 *s.* 8 *d.* for 3 weeks, at 6 *l.* per Cent. per Annum?

$$\begin{array}{r}
 \text{l.} \quad \text{l.} \quad \text{l.} \quad \text{s.} \quad \text{d.} \\
 \text{If } 100 \text{ --- } 6 \text{ --- } 321 \quad 16 \quad 8 \\
 \quad \quad \quad \quad \quad \quad \quad 6
 \end{array}$$

$$\begin{array}{r}
 19131 \quad 00 \quad 0 \\
 20
 \end{array}$$

*Answer, 19 *l.* 6 *s.* 2 *d.* $\frac{1}{4}$ for a year.*

$$\begin{array}{r}
 6120 \\
 12 \\
 \hline
 2140 \\
 4 \\
 \hline
 1160
 \end{array}$$

Then,

Then, If $52 \text{---} 19 \dots 6 \cdot 2\frac{1}{4} \text{---} 3$

$\frac{52}{57} \cdot 18 \cdot 6\frac{3}{4} (1 \cdot 2 \cdot 3\frac{1}{4})$ Answer.

$$\begin{array}{r} 5 \\ 20 \\ \hline 52)118(2 \end{array}$$

14
12

52) 174(3

18
4

52)75(1

23

Ex. 2. What interest is now due on a bond of 420 l. 10 s. commencing interest at 5 l. 10 s. per Cent. per Annum, July the 10th, 1744; this being March the 25th, 1746?

$\begin{matrix} l. & l. & s. & l. & s. \\ \text{If } 100 & \text{---} & 5 & 10 & \text{---} & 420 & 10 \end{matrix}$

_____5.
f. 2102 10

$$10\frac{1}{2} \quad 210 \quad 5$$

23112 15

20

23l. 2s. 6d. $\frac{1}{4}$
for a Year.

2155

12

6/60

4

2140

D 5

July	21
August	31
September	30
October	31
November	30
December	31
January	31
February	28
March	25

258

Then,

[70]

Then, *Days.* l. s. d. *Days.*
If 365—23 2 6½—258

20

462

12

5550

4

22202

258

177616

111010

44404

4

365)5728116(15693(1

365

12)3923(11

2078

1825

210)32(6

2531

16 l. 6 s. 11 d. $\frac{1}{4}$ for 258 days.

2190

23 l. 2 s. 6 d. $\frac{1}{2}$ for a year.

3411

39 l. 9 s. 5 d. $\frac{3}{4}$ total Interest.

3285

1266

1095

171

Another way to cast up Interest, when the principal has odd money in it, and the time, weeks, or days.

R U L E.

Bring your principal into its lowest denomination, and multiply it by the number of days; and if at 5 l.

per

per Cent. divide the product by 7300, the quotient gives the answer in the same denomination the principal was reduced into. *Note,* The number 7300 is found by multiplying 365 by 100, and dividing by 5.

EXAMPLES.

What's the interest of 432 *l.* 12 *s.* 6 *d.* for 8 days, at 5 *l. per Cent. per Annum*?

<i>l.</i>	<i>s.</i>	<i>d.</i>	
432	12	6	
	20		
8652			
	12		
103830			
	8		
	12		
73 00	8306	40	113
	73		
100			
	73		
276			
	219		
5740			
	4		
73 00	229	60	3
	219		
	10		

9s. 5d. $\frac{3}{4}$ Answer.

If you would cast up Interest at 6 *l. per Cent.* by this rule, you must add $\frac{1}{5}$ of the quotient to the interest produced at 5 *l. per Cent.* and so at other rates.

D 6 it

it is but adding or subtracting such and such parts of the quotient to or from what it produces at 5 l. For instance, suppose the last question done the common way, for an

E X A M P L E.

l. s.
420 10 *Principal.*
20

8410
623 *number of days.*

25230
16820
50460
s. d.

73|00)52934|30(717 8 $\frac{3}{4}$

511 s.
10 $\frac{1}{10}$ 71 9 $\frac{1}{4}$

129
73 78|9 6

564 l. l. 39:9:6 *Answer, 1 far-
thing more than
by the other way.*
511

5330
12

73|00)639|60(8

584

5560
4

73|00)222|40(3

219

340

Note,

Note, The Rate here being 5 *l.* 10*s.* and 10*s.* being $\frac{1}{10}$ of 5 *l.* there is therefore added $\frac{1}{10}$ of the quotient, or the amount, at 5 *l.* per Cent.

More Examples in Simple Interest.

At 6 *l.* per Cent. what will 589 *l.* come to in a year?

Answer, 35 *l.* 6*s.* 9*d.* $\frac{1}{2}$

What will the same sum, at the same rate, amount to in 8 years?

Answer, 282 *l.* 14*s.* 4*d.*

What's the interest of 7020 *l.* for 5 months, at 5 *l.* per Cent. per Annum?

Answer, 146 *l.* 5*s.*

What's a week's interest of 1000 *l.* at 7 *l.* per Cent?

Answer, 1 *l.* 6*s.* 11*d.*

What will the interest of 5429 *l.* 10*s.* 8*d.* come to in 20 days, at 5 *l.* per Cent?

Answer, 14 *l.* 17*s.* 6*d.*

What Interest is now due on a Navy-bill, commencing interest at 6 *l.* per Cent. per Annum, June the 4th 1743, this being January the 10th 1745, the principal being 836 *l.*?

Answer, 80 *l.* 7*s.* 10*d.*

COMPOUND INTEREST.

COMPOUND INTEREST, as was before said, arises both from the principal and interest; (*i. e.*) when interest on money becoming due, and not paid, the same rate is allowed on the unpaid interest, as was before on the principal.

There-

Therefore, to work sums in this rule, after having found the first year's interest, as before, add it to the principal, and find the interest of the sum; and so continue to add every year's produce, still accounting the sum a new principal.

E X A M P L E.

What will be the amount of 400 *l.* forborn 3 years and $\frac{1}{2}$, at 6 *l. per Cent. per Annum*, compound Interest?

l. *l.* *l.*
If 100 ——— 6 ——— 400 *Principal 1st year.*
6

l. 24|00

l. *l.* *l.*
If 100 ——— 6 ——— 424 *Principal 2d year.*
6

l. 25|44

20

s. 8|80

12

d. 9|60

4

2|40

[75]

l.
424
25 8 9½

l. l.
If 100 — 6 — 449 8 9½ Princip. 3d year.
6

l. 26196 12 9—
20

s. 19132
12

d. 3193
4

3172

l. s. d.
449 8 9½
26 19 3¾

l. l.
If 100 — 6 — 476 8 1¼ Princip. 4th year.
6

l. s. d. l. 28158 8 7½
28 11 8 20

M. —
6½ 14 5 10 s. 11168
476 8 1¼ 12

490 13 11¼ d. 8123
Total amount. 4

94

Or,

Or, more plain :

<i>Interest the 1st Year</i>	24	0	0—
2d—	25	8	9 $\frac{1}{2}$
3d—	26	19	3 $\frac{3}{4}$
$\frac{1}{2}$ Year—	14	5	10

Total Interest 90 13 11 $\frac{1}{4}$

Principal 400 0 0

Total Amount 490 13 11 $\frac{1}{4}$

More EXAMPLES.

What will 1000 *l.* amount to in 3 years, at 6 *l.* per Cent. per Annum, Compound Interest?

Answer, 1191 *l.* 0*s.* 3*d.* $\frac{3}{4}$.

At 5 *l.* 10*s.* per Cent. per Annum, Compound Interest, what will 560 *l.* 10*s.* 9*d.* amount to, forborn 5 Year and a half?

Answer, 752 *l.* 14*s.* 10*d.* $\frac{1}{2}$.

CHAP. VI.

Of REBATE, or DISCOUNT.

REBATE, or DISCOUNT, is the abating so much money on a debt paid before 'tis due, as might be gain'd again by the money receiv'd, if put out to interest at the same rate, and for the same time. So 100 *l.* present money, would discharge a debt of 106 *l.* due at a year to come; rebate being made at 6 *l.* per Cent. because 100 *l.* put out to interest for a year, at the said rate, would regain the 6 *l.*

To work Sums in this RULE.

First, find the interest of 100 *l.* for the time; then by a *Rule of Three* stating, of which the first number must be 100 *l.* with the interest found; the second number, the interest alone (or 100 *l.* alone, if you would find the present money); and the third, the debt, or sum propounded, you will find the answer.

EXAMPLES.

EXAMPLES.

Ex. 1. What's the rebate of 420 *l.* for a year, at 6 *l.* per Cent. per Annum?

$$\begin{array}{r}
 \text{If } \overset{\text{1.}}{106} \text{ — } \overset{\text{1.}}{6} \text{ — } \overset{\text{1.}}{420} \\
 \phantom{\text{If } 106 \text{ — } 6 \text{ — }} 6 \\
 \phantom{\text{If } 106 \text{ — } 6 \text{ — }} \text{— } \text{1.} \\
 106)2520(23 \\
 \underline{212} \\
 \cdot 400 \\
 318 \\
 \phantom{\cdot } \text{—} \\
 \phantom{\cdot } 82 \\
 \phantom{\cdot } \phantom{\cdot } 20
 \end{array}$$

Ans^w. 23 *l.* 15 *s.* 5 *d.* $\frac{1}{2}$

$$\begin{array}{r}
 \text{106)1640(15} \\
 \underline{106} \\
 \cdot 580 \\
 \cdot 530 \\
 \text{—} \\
 50 \\
 12 \\
 \text{— } \text{d.} \\
 106)600(5 \\
 \underline{530} \\
 \cdot 70 \\
 4 \\
 \text{—} \\
 106)280(2 \\
 \underline{212} \\
 \cdot 68
 \end{array}$$

Ex. 2.

Ex. 2. What present money will discharge a debt of 732 l. due at 3 months, rebate being made at 6 l. per Cent. per Annum ?

l. l.

6 Interest for a Year of 100

Months —

3 $\frac{1}{4}$ 10

l. s. l.

Then, If 101 10 fall to 100, what will 732 ?

2

2

203

203)146400(721

1421

430

406

: 240

203

: 37

20

203)740(3

609

131

12

203)1572(7

1421

: 151

4

203)604(2

406

198

Answer, 721 l. 3s. 7d. $\frac{1}{2}$

Ex. 3.

Ex. 3. What rebate must be allow'd on a Bill of Exchange for 85 l. 10 s. due *September* the 8th, this being *July* the 4th, rebate being made at 5 l. per Cent. per Annum?

	Days.	l.	Days.
	If 365	5	66
July	27		5
Aug.	31		
Sept.	8		330
			20
	66		s. d.
			365)6600(18 0 $\frac{3}{4}$
			365
			2950
			2920
			30
			12
			360
			4
			365)1440(3
			1095
			345

Then,

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>s.</i>	<i>d.</i>	<i>l.</i>	<i>s.</i>
Then, If	100	18	0 $\frac{1}{4}$	18	0 $\frac{1}{4}$	85	10
	20			12		20	
	20	18		216		17	10
	12			4		12	
	24	216		867		20	520
	4					4	
	96	867				82	080
						867	
						57	4560
						49	2480
						65	6640
						4	
	96	867		71	163	360	(734(2
				67	8069		
					12	183	(3
					33	5646	
					29	0601	15 s. 3 d. $\frac{1}{2}$
					45	0450	
					38	7468	
					62	982	

Answer, 15 s. 3 d. $\frac{1}{2}$

Ex. 4. A Bill of Exchange for 250 *l.* is dated at *Amsterdam*, June the 14th, N. S. at usance is accepted, and payment offer'd the 15th ditto, O. S. what must be then receiv'd, rebate being made at 6 *l.* per Cent. per Annum?

Note, New Style is 11 days before Old Style.

Usance signifies the usual time of payment of bills between one country and another; which between *England* and *Holland* is one month; double Usance, two months, &c. Note also, Merchants of *London* generally

rally allow 3 days beyond the time appointed, calling them 3 days of grace. Therefore the bill being dated *June* the 14th N. S. or the 3d O. S. and accepted at usance, or a month, it becomes due *July* the 3d, O. S. but 3 days of grace being allow'd, makes it the 6th. So if the money is paid the 15th of *June* O. S. rebate must be made for 21 days. Therefore.

<i>Days.</i>	<i>l.</i>	<i>Days.</i>
If 365	6	21
		6
		126
		20
		365)2520(6
		2190
		330
		12
		d.
		365)3960(10
		365
		310
		4
		365)1240(3
		1095
		145

facit 6s. 10 d. ¾

<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>l.</i>	<i>l.</i>
If 100	6	101	100	250
	20			20
2006				5000
12				12
24082				60000
4				4
96331			96331	24000000(249
				192662
			473380	
			385324	
			880560	
			866979	
			13581	
			20	
			96331	271620(2
				192662
			78958	
			12	
			96331	947496(9
				866979
			80517	
			4	
			96331	322068(3
				288993
			33075	

Answer, 249 *l.* 2 *s.* 9 *d.* $\frac{3}{4}$

More EXAMPLES.

What's the rebate of 100 for $\frac{1}{2}$ a year, at 6 l. per Cent. per Annum?

Answer, 2 l. 18 s. 3 d.

What's the rebate of 538 l. 10 s. for 5 months, at 5 l. per Cent. per Annum?

Answer, 10 l. 19 s. 9 d. $\frac{1}{2}$

What present money is a debt of 629 l. due at 3 weeks, worth, rebate being made after the rate of 6 l. 10 s. per Cent. per Annum?

Answer, 626 l. 13 s.

What present money will discharge 10000 l. due at 5 days, rebate being made at 8 l. per Cent. per Annum?

Answer, 9989 l. 1 s. 5 d. $\frac{3}{4}$.

CHAP. VII.

Of EQUATION of PAYMENTS.

THE design of this rule is, when several sums are due at several times, to find a mean time for paying the whole debt.

The common way to do which is,

Multiply each sum by its respective time, and add all the products together, dividing their total by the whole debt; the quotient is accounted the mean time of paying the whole debt.

EXAMPLE.

A. owes to B. 40*l.* to pay at 3 months, 60*l.* at 5 months, and 100*l.* at 10 months; at what time may the whole debt be paid together, without injury to either?

<i>l.</i>	<i>l.</i>	<i>l.</i>	<i>l.</i>
40	40	60	100
60	3	5	10
100	<u> </u>	<u> </u>	<u> </u>
<u> </u>	120	300	1000
200			300
			120
			<u> </u>

2100)14120(

Answer, 7 months.

E

Note,

Note. The times of payment, if not so given, must be reduced into one name.

But those who are more exact take the pains to work these sums by the following rule.

First, they find the present worth of every particular sum; then find in what time to come the sum of these present worths will be increased to the total of all the particular sums payable at the several times to come; and that is the true equated time for payment of the whole.

001	002	003	004
005	006	007	008
009	010	011	012
013	014	015	016
017	018	019	020
021	022	023	024
025	026	027	028
029	030	031	032
033	034	035	036
037	038	039	040
041	042	043	044
045	046	047	048
049	050	051	052
053	054	055	056
057	058	059	060
061	062	063	064
065	066	067	068
069	070	071	072
073	074	075	076
077	078	079	080
081	082	083	084
085	086	087	088
089	090	091	092
093	094	095	096
097	098	099	100
101	102	103	104
105	106	107	108
109	110	111	112
113	114	115	116
117	118	119	120
121	122	123	124
125	126	127	128
129	130	131	132
133	134	135	136
137	138	139	140
141	142	143	144
145	146	147	148
149	150	151	152
153	154	155	156
157	158	159	160
161	162	163	164
165	166	167	168
169	170	171	172
173	174	175	176
177	178	179	180
181	182	183	184
185	186	187	188
189	190	191	192
193	194	195	196
197	198	199	200
201	202	203	204
205	206	207	208
209	210	211	212
213	214	215	216
217	218	219	220
221	222	223	224
225	226	227	228
229	230	231	232
233	234	235	236
237	238	239	240
241	242	243	244
245	246	247	248
249	250	251	252
253	254	255	256
257	258	259	260
261	262	263	264
265	266	267	268
269	270	271	272
273	274	275	276
277	278	279	280
281	282	283	284
285	286	287	288
289	290	291	292
293	294	295	296
297	298	299	300
301	302	303	304
305	306	307	308
309	310	311	312
313	314	315	316
317	318	319	320
321	322	323	324
325	326	327	328
329	330	331	332
333	334	335	336
337	338	339	340
341	342	343	344
345	346	347	348
349	350	351	352
353	354	355	356
357	358	359	360
361	362	363	364
365	366	367	368
369	370	371	372
373	374	375	376
377	378	379	380
381	382	383	384
385	386	387	388
389	390	391	392
393	394	395	396
397	398	399	400
401	402	403	404
405	406	407	408
409	410	411	412
413	414	415	416
417	418	419	420
421	422	423	424
425	426	427	428
429	430	431	432
433	434	435	436
437	438	439	440
441	442	443	444
445	446	447	448
449	450	451	452
453	454	455	456
457	458	459	460
461	462	463	464
465	466	467	468
469	470	471	472
473	474	475	476
477	478	479	480
481	482	483	484
485	486	487	488
489	490	491	492
493	494	495	496
497	498	499	500
501	502	503	504
505	506	507	508
509	510	511	512
513	514	515	516
517	518	519	520
521	522	523	524
525	526	527	528
529	530	531	532
533	534	535	536
537	538	539	540
541	542	543	544
545	546	547	548
549	550	551	552
553	554	555	556
557	558	559	560
561	562	563	564
565	566	567	568
569	570	571	572
573	574	575	576
577	578	579	580
581	582	583	584
585	586	587	588
589	590	591	592
593	594	595	596
597	598	599	600
601	602	603	604
605	606	607	608
609	610	611	612
613	614	615	616
617	618	619	620
621	622	623	624
625	626	627	628
629	630	631	632
633	634	635	636
637	638	639	640
641	642	643	644
645	646	647	648
649	650	651	652
653	654	655	656
657	658	659	660
661	662	663	664
665	666	667	668
669	670	671	672
673	674	675	676
677	678	679	680
681	682	683	684
685	686	687	688
689	690	691	692
693	694	695	696
697	698	699	700
701	702	703	704
705	706	707	708
709	710	711	712
713	714	715	716
717	718	719	720
721	722	723	724
725	726	727	728
729	730	731	732
733	734	735	736
737	738	739	740
741	742	743	744
745	746	747	748
749	750	751	752
753	754	755	756
757	758	759	760
761	762	763	764
765	766	767	768
769	770	771	772
773	774	775	776
777	778	779	780
781	782	783	784
785	786	787	788
789	790	791	792
793	794	795	796
797	798	799	800
801	802	803	804
805	806	807	808
809	810	811	812
813	814	815	816
817	818	819	820
821	822	823	824
825	826	827	828
829	830	831	832
833	834	835	836
837	838	839	840
841	842	843	844
845	846	847	848
849	850	851	852
853	854	855	856
857	858	859	860
861	862	863	864
865	866	867	868
869	870	871	872
873	874	875	876
877	878	879	880
881	882	883	884
885	886	887	888
889	890	891	892
893	894	895	896
897	898	899	900
901	902	903	904
905	906	907	908
909	910	911	912
913	914	915	916
917	918	919	920
921	922	923	924
925	926	927	928
929	930	931	932
933	934	935	936
937	938	939	940
941	942	943	944
945	946	947	948
949	950	951	952
953	954	955	956
957	958	959	960
961	962	963	964
965	966	967	968
969	970	971	972
973	974	975	976
977	978	979	980
981	982	983	984
985	986	987	988
989	990	991	992
993	994	995	996
997	998	999	1000

C H A P. VIII.

Of FELLOWSHIP, or COMPANY.

FELLOWSHIP, or COMPANY, is when two or more join their stocks, and trade together, dividing their gain or loss proportionably.

FELLOWSHIP is either with, or without time. Questions without time, are work'd by this proportion.

As the whole stock,
To the whole gain, or loss:
So is each man's particular stock,
To his particular share.

E X A M P L E S.

A. B. and C. make a joint-stock; A. puts in 460 l. B. 510 l. and C. 480 l. they gain 340 l. what part of it belongs to each?

	<i>l.</i>
<i>A.</i> _____	460
<i>B.</i> _____	510
<i>C.</i> _____	480
<i>Total stock,</i>	1450

1st. Then, If $\begin{array}{r} \text{I.} \\ 145 \end{array} | 0 \text{---} \begin{array}{r} \text{I.} \\ 340 \end{array} \text{---} \begin{array}{r} \text{I.} \\ 46 \end{array} | 0$
 $\begin{array}{r} \\ \\ 340 \end{array}$

2d. If $\overset{l.}{145} | \overset{l.}{0} - \overset{l.}{34} \circ - 51 | \underset{34^\circ}{0}$

$$\begin{array}{r} 2040 \\ 153 \\ \hline \end{array}$$

145)17340(119
145

• 284
145

1390
1305

85
20

— s.
(145)1700(11
145

250
145

105
12

~~_____~~d.
[145)1260(8
1160

• 100
4

$$\begin{array}{r} 145 \overline{) 400} (2 \\ \underline{290} \\ 110 \end{array}$$

1840
138

145) 15640 (107
145

• 1140
1015

125
2

———— s.
145)2500(17
145

105
101

• • 2
125 1

$$\begin{array}{r} \text{---}d. \\ 145 \overline{)420} (2 \\ \underline{290} \end{array}$$

13

145)520(3
435

8

3d. And, If 145|0 ——— 340 ——— 48|0
340

1920

144

145)16320(112

145

• 182

145

• 370

290

• 80

20

s.

145)1600(11

145

• 150

145

• 5

12

145)290(2

60

4

145)240(1

145

• 95

	<i>l.</i>	<i>—l.</i>	<i>d.</i>	<i>Rem.</i>
A's Share is	107	17	2 $\frac{3}{4}$	85
B's Share is	119	11	8 $\frac{1}{2}$	110
C's Share is	112	11	0 $\frac{1}{4}$	95
Value of the Remainder			$\frac{1}{2}$	
				145)290(2
The whole } gain.	340	00	0	290

• 0

And thus are these Sums proved.

More E X A M P L E S.

Three persons make a joint-stock ; *A.* puts in 565 *l.* *B.* 278 *l.* and *C.* 629 *l.* they gain 1000 *l.* and what is each of their shares ?

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>Remainders.</i>
<i>Answer,</i> { <i>A.</i>	383	16	7 $\frac{1}{2}$	384
{ <i>B.</i>	188	17	2	512
{ <i>C.</i>	427	6	2 $\frac{1}{4}$	576

D. E. and *F.* trade together ; *D.* puts in 230 *l.* *E.* 529 *l.* and *F.* 344 *l.* 10 *s.* their whole gain amounts to 520 *l.* what is that to each ?

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>Remainders.</i>
<i>Answer,</i> { <i>D.</i>	108	7	7 $\frac{3}{4}$	271
{ <i>E.</i>	249	5	7	844
{ <i>F.</i>	162	6	9	1092

A. B. and *C.* make a bank of 3256 *l.* whereof *A.* puts in 1026 *l.* *B.* 985 *l.* and *C.* the rest ; by misfortune they lose 2000 *l.* what part of it must each bear ?

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>Remainders.</i>
<i>Answer,</i> { <i>A.</i>	630	4	5	928
{ <i>B.</i>	605	0	8 $\frac{3}{4}$	1240
{ <i>C.</i>	764	14	10	1088

FELLOWSHIP with TIME.

FELLOWSHIP with TIME is thus work'd ; multiply each man's stock by the respective time he puts it in for, and add all the products, the total of which must be your first number through all the statings ; the gain,

gain, or loss, the second (as before); and each man's particular stock, multiply'd by its time, the third.

Note, All the particular times (if not so given) must be reduced into one Name, (*i. e.*) all years, all months, all weeks, or all days, &c.

The reason of multiplying the several stocks by the times they are put in for, will appear from the following considerations. *First*, suppose *A.* and *B.* put into trade each 100 *l.* for 1 month, then certainly their gain, whatever it be, must be equally divided betwixt them. Again, suppose *A.* puts in 100 *l.* and *B.* 200 *l.* both for the same time; 'tis as plain that *B.*'s share of the gain must be twice as much as *A.*'s, because his stock is double: or if each put in 100 *l.* but for different times, *viz.* *A.* for one month, and *B.* for 2; then as *B.*'s stock lies twice as long as *A.*'s, so his gain, as before, must be twice as great. *Lastly*, suppose *A.* to put in 100 *l.* for 1 month, and *B.* 200 *l.* for 2 months; then since his stock is not only double, but the time he leaves it in double too, his share of the profits must become 4 times greater than *A.*'s. And so in all other cases.

E X A M P L E.

A. put into company 560 *l.* for 8 months, *B.* 279 *l.* for 10 months, and *C.* 735 *l.* for 6 months; they gain'd 1000 *l.* What share of it must each have?

<i>l.</i>	<i>l.</i>	<i>l.</i>
560	279	735
8	10	6
<i>A.</i> —	<i>B.</i> —	<i>C.</i> —
4480	2790	4410
		2790
		4480
		11680

[92]

1/β. Then, If $\frac{1}{1168} \frac{1}{10} - \frac{1}{1000} - \frac{1}{448} \frac{1}{10}$
 $\frac{1000}{1000}$

$1168)448000(383$
 3504

$\cdot 9760$
 9344

$\cdot 4160$
 3504

$\cdot 656$
 20

$1168)13120(11$
 1168

$\cdot 1440$
 1168

$\cdot 272$
 12

$1168)3264(2$
 2336

$\cdot 928$
 4

$1168)3712(3$
 3504
 $\cdot 208$

[93]

2d. If $\overset{L}{1168} \overset{L}{|0} \overset{L}{-1000} \overset{L}{-279} \overset{L}{|0}$
 $\quad \quad \quad 1000$

$\quad \quad \quad \underline{\quad \quad \quad} \overset{L}{1168} 279000 \overset{L}{(238}$
 $\quad \quad \quad 2336$

$\quad \quad \quad \underline{\quad \quad \quad} \overset{L}{4540}$
 $\quad \quad \quad 3504$

$\quad \quad \quad \underline{\quad \quad \quad} 10360$
 $\quad \quad \quad 9344$

$\quad \quad \quad \underline{\quad \quad \quad} 1016$
 $\quad \quad \quad 20$

$\quad \quad \quad \underline{\quad \quad \quad} \overset{s}{1168} 20320 \overset{s}{(17}$
 $\quad \quad \quad 1168$

$\quad \quad \quad \underline{\quad \quad \quad} \overset{s}{8640}$
 $\quad \quad \quad 8176$

$\quad \quad \quad \underline{\quad \quad \quad} \overset{s}{464}$
 $\quad \quad \quad 12$

$\quad \quad \quad \underline{\quad \quad \quad} \overset{d}{1168} 5568 \overset{d}{(4}$
 $\quad \quad \quad 4672$

$\quad \quad \quad \underline{\quad \quad \quad} \overset{d}{896}$
 $\quad \quad \quad 4$

$\quad \quad \quad \underline{\quad \quad \quad} 1168 3584 \overset{d}{(3}$
 $\quad \quad \quad 3504$
 $\quad \quad \quad \underline{\quad \quad \quad} \overset{d}{80}$

			<i>l.</i>	<i>l.</i>	<i>l.</i>
3d. If			1168 0	1000	441 0
					1000
					<i>l.</i>
					1168)441000(377
					3504
	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>Rem.</i>	
A's Share is	383	11	2 $\frac{3}{4}$	208	
B's Share is	238	17	4 $\frac{3}{4}$	80	9060
C's Share is	377	11	4 $\frac{1}{4}$	880	8176
Value of the Remainders $\frac{1}{4}$			1168)	1168(1	8840
					8176
			1000 00 0—	...	0
The whole gain.					664
					20
					<i>s.</i>
					1168)13280(11
					1168
					1600
					1168
					432
					12
					<i>d.</i>
					1168)5184(4
					4672
					512
					4
					1168)2048(1
					1168
					880

More EXAMPLES.

A. B. and C. agreeing to trade together, A. puts in 529 l. for 4 months; B. 329 l. for 7 months; and C. 900 l. for 2 months; by their trade they gain 540 l. What part of it belongs to each?

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>Remainders.</i>
<i>Answer,</i> { <i>A.</i>	183	14	8	2304
{ <i>B.</i>	199	19	5	1332
{ <i>C.</i>	156	5	10 $\frac{3}{4}$	2583

Three persons make a joint-stock; *A* puts in 400 pieces of holland, each containing 96 yards, at 5*s.* 6*d.* per ell *Flemish* for 5 months; *B.* 600 guineas for 8 months; and *C.* 1800 *l.* for 4 months, but at 3 months end takes out 800 *l.* They gain in all 1000 *l.* How much is each of their shares?

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>Remainders.</i>
<i>Answer,</i> { <i>A.</i>	860	4	3 $\frac{1}{2}$	3996
{ <i>B.</i>	61	11	8	1920
{ <i>C.</i>	78	4	0 $\frac{1}{4}$	2568

CHAP. IX.

Of ALLIGATION.

ALLIGATION is the compounding many simples into one mass, according to any requir'd price, or proportion.

This rule is usually divided into two parts, distinguish'd by the names of *Medial* and *Alternate*.

ALLIGATION MEDIAL.

Alligation Medial answers such questions as having the quantities and prices of the several simples given, only demand the mean rate of any particular part of the composition.

R U L E.

As the sum of all the simples,
To their total value:
So is any quantity of the mixture,
To the price thereof.

E X A M P L E S.

(*Ex. 1st.*) A grocer would mix 18 lb. of currans at 5 *d. per lb.* and 29 lb. at 6 *d. per lb.* with 12 lb. at 8 *d. per lb.* At what rate may he sell a pound of the mixture?

First,

First, place the quantities and their values, as below.

lb.	d.		s.	d.
18	at 5	per lb. comes to	7	6
20	at 6	_____ to	14	6
12	at 8	_____ to	8	0

Sum of all the Sim. 59

l. 1 10 0 tot. val.

Then, If 59 comes to 1 10 what will 1?

20

—

30

12

—d.

56)360(6 Answer.

354

—

• 6

Ex. 2. A grocer would with 12 C. of sugar, at 3 l. per C. mix 3 C. at 50 s. per C. and 8 C. at 45 s. per C. What will 500 of this mixture be worth?

The quantities and their values, placed as before, will stand thus:

C.		l.	s.
12	at 3 l. per C. is	36	00
3	at 50 s. per C. is	7	10
8	at 45 s. per C. is	18	00

Sum of the Simples 23

61 10 tot. val.

Then,

[98]

Then, $\overset{C.}{\text{If } 23} \text{ --- } \overset{l.}{61} \overset{s.}{10} \text{ --- } \overset{C.}{5}$

20

1230

5

23)6150(2617

46

13 l. 7 s. 4 d $\frac{1}{2}$ facit.

155

138

170

161

9

12

23)108(4

92

16

4

23)64(2

46

18

The proof of this is as in the *Rule of Three*, only by reversing the proportion.

So the first example may be prov'd by this back-
stating,

lb. d. lb.

If 1 --- 6 --- 59

And the last by this ;

C. l. s. d. C.

If 5 --- 13 7 4 $\frac{1}{2}$ --- 23

remembering in both to take in the remainders.

Mori

More E X A M P L E S.

If a vintner mingleth 13 gallons of canary, at 6*s.* 8*d.* per gallon, with 20 gallons of white-wine, at 5*s.* per gallon, and 10 gallons of cyder, at 3*s.* per gallon, at what rate must he sell a quart of the mixture?

Answer, 1*s.* 3*d.*

If with 40 bushels of corn, at 4*s.* per bushel, there are mix'd 10 bushels at 6*s.* per bushel; 30 bushels at 5*s.* per bushel, and 20 bushels at 3*s.* per bushel, what will 10 bushels of the mistling be worth?

Answer, 2*l.* 3*s.*

A L L I G A T I O N A L T E R N A T E.

Alligation Alternate teacheth what quantities of several simples (whose values are given) must be mixed, that the composition may bear a price propounded.

This sort of *Alligation* has three varieties.

V A R I E T Y I.

When the price of each simple is exprest, but no quantity given, and it is requir'd how much of each simple must be mix'd, to sell any part of the composition at a mean price propounded?

In this variety you have only to link the several extremes rightly together, and to find the true differences betwixt them and the mean; for those differences are the quantities sought.

E X A M P L E.

A vintner would mix four sorts of wine, of 12*d.* of 18*d.* of 24*d.* and of 26*d.* per quart; what quantity of each must he take, to sell the mixture at 20*d.* per quart?

To

To place your numbers, and link the extremes, observe these rules.

R U L E S.

1st. Set down the several given prices (which must always be of one name) in a column, one under another; then, towards your left hand, draw a line of connection; and on the other side of the said line place the mean rate.

Thus then will stand the given numbers of the given example :

$$20 \left\{ \begin{array}{l} 12 \\ 18 \\ 24 \\ 26 \end{array} \right.$$

2^d. Link them two and two together, always observing to join a greater and a less than the mean; and against each extreme place the difference betwixt the mean and its yoke-fellow :

$$\begin{array}{rcc} & \text{Thus,} & \\ 20 \left\{ \begin{array}{l} 12 \\ 18 \\ 24 \\ 26 \end{array} \right. & \begin{array}{|l} \hline \\ \hline \\ \hline \\ \hline \end{array} & \begin{array}{l} 4 \\ 6 \\ 8 \\ 2 \end{array} \end{array}$$

So the difference betwixt the extreme 12, and 20 the mean, being 8, is placed against 24, its yoke-fellow; and between 18 and 20 being 2, is placed against 26: so also the difference betwixt 24 and 20 being 4, is placed against 12; and 6, the difference betwixt 26 and 20, is set against 18.

Therefore, if he mixes 4 quarts at 12*d.* 6 at 18*d.* 8 at 24*d.* and 2 at 26*d.* *per* quart, he may sell the mixture at 20*d.* *per* quart.

The proof of which is easy. For the sum of the differences valued at the mean rate, will be found equal to the amount of the particular differences at their given prices.

For

For 20 quarts, the sum of the differences at 20 *d.* per quart, is 400 pence; and 4, 6, 8, 2 quarts, the particular differences, at 12, 18, 24, 26 pence per quart, will come to the same sum.

But note, as many different ways as the extremes may be combin'd, so many different answers may be given to the question, yet all true.

So the same example being combin'd, thus :

$$20 \left\{ \begin{array}{l|l} 12 & 6 \\ 18 & 4 \\ 24 & 2 \\ 26 & 8 \end{array} \right\} \text{Answ.} \quad 20 \left\{ \begin{array}{l|l|l} 12 & 4.6 & 10 \\ 18 & 4.6 & 10 \\ 24 & 8.2 & 10 \\ 26 & 8.2 & 10 \end{array} \right\} \text{Answ.}$$

gives those different, but true answers, as may be prov'd by the same rule.

In the last of these examples the numbers being doubly combin'd (*i. e.*) each with two others, make each have two differences set against it; for 12, the first extreme, being link'd both with 24 and 26, must have both their differences, *viz.* 4 and 6, placed against it; and so the rest. Which double difference must be added, and placed as above.

But when in a question there is but one extreme less, or but one greater than the mean; such a question will admit of but one answer, that single extreme being to be link'd with all the rest.

EXAMPLES.

A grocer would mix sugars, at 5 *d.* 6 *d.* and 10 *d.* per lb. so as to sell the compound at 8 *d.* per lb. What quantity of each must he take?

$$8 \left\{ \begin{array}{l|l|l} 5 & 2 & 2 \\ 6 & 2 & 2 \\ 10 & 3.2 & 5 \end{array} \right.$$

The numbers being plac'd, link'd, and differenced, (as above) according to the foregoing rules, it appears, to sell his sugar at 8 *d.* per lb. he must mix 2 lb. at 5 *d.* 2 lb. at 6 *d.* and 5 lb. at 10 *d.* per lb.

The

The reason of these combinations, and the alternate placing of their differences, will appear from this plain consideration, *viz.* That thereby whatever is lost upon the quantity sold, whose given price exceeds the mean, is gain'd again upon the quantity, whose given price is less than the mean.

More E X A M P L E S.

A tobacconist would mix tobacco of 2 s. and 1 s. 6 d. and 1 s. 3 d. *per lb.* so as to sell the compound at 1 s. 3 d. *per lb.* What quantity of each must he take?

$$\text{Answer, } \left\{ \begin{array}{l} \text{lb.} \quad \text{s.} \quad \text{d.} \\ 7 \text{ at } 2 \quad 0 \\ 4 \text{ at } 1 \quad 6 \\ 4 \text{ at } 1 \quad 3 \end{array} \right\} \text{ per lb.}$$

Several sorts of tea, *viz.* of 13 s. 18 s. 26 s. and 30 s. *per lb.* are to be so mixed, that the compound may be worth 22 s. *per lb.* what quantity of each must there be?

$$\text{Answer, } \left\{ \begin{array}{l} \text{lb.} \quad \text{s.} \\ 8 \text{ at } 13 \\ 4 \text{ at } 18 \\ 4 \text{ at } 26 \\ 9 \text{ at } 30 \end{array} \right\} \text{ per lb.}$$

V A R I E T Y 2.

Here we have the price of all the simples, and the quantity of one given, to find the several quantities of the rest: so that one measure, or quantity, may be sold at a mean rate propounded.

When the numbers have been set down, and their several differences found by the foregoing rule, you must proceed by this proportion.

As the difference of that number whose quantity is given,

To the rest of the differences one after another:

So the quantity given,

To the several quantities sought.

E X A M P L E.

E X A M P L E.

A distiller would with 40 gallons of *French* brandy, at 12 s. per gallon, mix *English* brandy at 7 s. per gallon, and spirits at 4 s. per gallon: how much of the last two sorts must be added, to sell the whole at 8 s. per gallon?

$$8 \left\{ \begin{array}{l} 12 \overline{) 1} \\ 7 \overline{) 4} \\ 4 \overline{) 4} \end{array} \right. \begin{array}{l} 1.4 | 5 \\ 4 | 4 \\ 4 | 4 \end{array}$$

Now it is plain, that were there but 5 gallons of the *French* brandy, there must also be but 4 gallons of each of the other liquors; but since there are to be 40 gallons of the *French* brandy, say,

As 5, the difference against its price,

To 4, the difference against the price of the *English* brandy:

So is 40, the given quantity of the *French*,

To the quantity of *English* sought:

Which by this proportion will be found to be 32.

Then again,

As the first difference 5,

To 4, the difference against the price of the spirits:

So 40, the said given quantity,

To the quantity of spirits sought.

Which, because their differences are alike, will be the same as of the *English* brandy, viz. 32.

This variety has the same proof as the former.

More E X A M P L E S.

How much tea at 14 s. per lb. and of 18 s. per lb. must be mix'd with 8 lb. at 24 s. per lb. so as to allow the lb. at 20 s.?

$$\text{Answer, } \left\{ \begin{array}{l} 8 \text{ lb. at } 24 \text{ s.} \\ 4 \text{ lb. at } 18 \text{ s.} \\ 4 \text{ lb. at } 14 \text{ s.} \end{array} \right.$$

I would with 21 lb. of snuff, at 8 s. per lb. mix so much of 4 s. per lb. and so much of 5 s. per lb. that I might

might afford to sell the mixture at 6s. per lb. What quantity of each must I take?

Answer, $\left\{ \begin{array}{l} 21 \text{ at } 8 \\ 14 \text{ at } 4 \\ 14 \text{ at } 5 \end{array} \right\} \text{ per lb.}$

VARIETY 3.

In this variety the particular prices of each simple being given, as also the mean rate of the compound, it is requir'd how much of each sort must be taken, to make up the quantity propounded?

Which (after the work of the first variety) is perform'd by this proportion:

As the sum of the differences,
To the total quantity:
So each particular difference,
To its particular quantity.

EXAMPLE.

A druggist hath 4 sorts of green tea, at 5s. 6s. 8s. and 9s. per lb. and would make up a quantity of 87 lb. to sell at 7s. per lb. how much of each must he take?

See the work.

lb.	s.	lb.	lb.	lb.
If 6 — 87 — 2				
7	{	5 —	2 — 29 at 5 per lb.	2
		6 —	1 — 14½ at 6 per lb.	— lb.
		8 —	1 — 14½ at 8 per lb.	6) 174 (29
		9 —	2 — 29 at 9 per lb.	—
			6	0

lb.	lb.	lb.
If 6 — 87 — 1		
1		
— lb.		
6) 87 (14½		
—		
3		

More

More *EXAMPLES*.

A vintner hath 3 sorts of wine, of 6*s.* 8*s.* and 10*s.* *per* gallon, of all which he would compose a quantity of 120 gallons, to sell at 7*s.* *per* gallon: what quantity must there be of each?

$$\text{Answer, } \left\{ \begin{array}{l} \text{gall.} \quad \text{s.} \\ 80 \text{ at } 06 \\ 20 \text{ at } 08 \\ 20 \text{ at } 10 \end{array} \right\} \text{ per Gall.}$$

A grocer having sugars of 5*d.* 6*d.* 8*d.* 10*d.* and 11*d.* *per* lb. would make a mixture of 3 C. *wt.* to sell at 7*d.* *per* pound; how much of each must he take?

$$\text{Answer, } \left\{ \begin{array}{l} \text{lb.} \quad \text{d.} \\ 28 \text{ at } 05 \\ 196 \text{ at } 06 \\ 56 \text{ at } 08 \\ 28 \text{ at } 10 \\ 28 \text{ at } 11 \end{array} \right\} \text{ per lb.}$$

C H A P. X.

Of POSITION, or the Rule of FALSE.

THIS rule has not the name of *False*, from its being in itself really erroneous, but only because that by the help of false supposed numbers we find the truth.

It is usually divided into two parts, *single* and *double*.

Single POSITION.

IN *Single Position* we make but one supposition, with which, working as with a true number, we regulate the result by this proportion, *viz.*

As the total arising from the error,
To the true total :
So the supposed part,
To the true one.

E X A M P L E S.

1st. *A. B. and C. designing to buy a quantity of lead, to the value of 140*l.* agree that B. shall pay as much again as A. and C. as much again as B. What then must each pay ?*

Now,

Now, suppose *A.* to pay 10*l.* then *B.* must pay 20*l.* and *C.* 40*l.* the total of which is 70*l.* but should be 140*l.* Therefore,

If 70*l.* should be 140*l.* what should 10*l.* be?

l.

Answer, 20 for *A*'s share ;
which doubled, makes 40 for *B*'s share ;
and that again doubled, gives 80 for *C*'s share.

The total of which is 140

2. Suppose the sum of 20*l.* were to be paid by *A.* *B.* and *C.* thus, *A.* $\frac{1}{2}$, *B.* $\frac{1}{3}$, *C.* $\frac{1}{4}$; what must each pay according to that proportion ?

	<i>l.</i>	<i>s.</i>	<i>d.</i>
First, the $\frac{1}{2}$ of 20 <i>l.</i> being	10	00	0
the $\frac{1}{3}$	6	13	4
and the $\frac{1}{4}$	5	00	0

The total will be 21 13 4

But it should have been but 20*l.* therefore the parts found should be thus regulated:

	<i>l.</i>	<i>s.</i>	<i>d.</i>		<i>l.</i>	<i>s.</i>	<i>d.</i>
<i>l.</i> <i>s.</i> <i>d.</i>	10	00	0	}	9	04	$7\frac{1}{4}$
If 21 13 4 fall to 20,	06	13	4		6	03	$0\frac{3}{4}$
what will	05	00	0		4	12	$3\frac{1}{2}$

which, with the addition of $\frac{1}{2}$, makes just 20 00 0—

found by dividing the total of $\left\{ \begin{array}{l} 28 \\ 36 \\ 40 \end{array} \right.$
the remainders—

by the common divisor 52)104(2

0

More E X A M P L E S.

A gentleman bought a chaise, horse, and harness, for 50*l.* the horse came to twice the price of the harness,

harness, and the chaise to twice as much as both horse and harness; what did he give for each?

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>Remainders.</i>
<i>the harness</i>	05	11	$1\frac{1}{4}$	15
<i>the horse</i>	11	02	$2\frac{1}{2}$	30
<i>the chaise</i>	33	06	8	

A. B. and *C.* disputing of their age, *A.* affirms he is as old as *B.* and $\frac{1}{2}$ as old as *C.* and *B.* says, that he is $\frac{3}{4}$ as old as *C.* upon which *C.* says, he is sure both their ages, added to his, will make 110 years. I demand the age of each.

$$\text{Answer, } \begin{cases} A. 45 \frac{5}{8} \\ B. 27 \frac{3}{8} \\ C. 36 \frac{3}{8} \end{cases}$$

Double POSITION.

IN *Double POSITION* two suppositions are required, which (if both prove false) are, with their errors, to be thus disposed.

Set down both your suppositions, and against each of them their respective errors, marked thus +, if too much; or thus—, if too little.

Then multiplying them cross-ways, that is, the first supposition by the second error, and the second supposition by the first error; if both the errors are alike, *i. e.* both too little, or both too much, subtract the lesser product from the greater, and divide the remainder by the difference of the errors. But if the errors are unlike, *i. e.* one too little, and the other too much, the sum of these products must be divided by the sum of the errors: the quotient will give the true number, exactly answering all the demands of the question.

Or the suppositions and their errors being placed as before, work by this proportion, as a general rule, *viz.*

As

As the difference of the errors, if alike,
Or their sum, if unlike,

To the difference of the suppositions:

So either error to a fourth number,

which accordingly added to, or subtracted from the
supposition against it, will answer the question.

E X A M P L E S.

1st *A. B. and C. playing at cards, the money stak'd
was 324 crowns; but disagreeing, each seiz'd as many
as he could; A. got a certain number, B. as many as
A. and 15 over; C. got a fifth part of both their sums
added together: how many had each?*

First, *suppose A. got 50*
then B's share will be 65
and C's ————— 23
—————

All which added, make but 138, which is 186 too few.

Then *supposing again, that A got 80*
B's share will be 95
and C's ————— 35
—————

The total of which is still but 210, too few by 114.

Therefore, setting down the suppositions, and their
errors (mark'd as before directed) against them, they
will stand thus:

<i>Suppositions.</i>	<i>Errors.</i>
50 —————	186
80 —————	114

then multiplied cross-ways, the products will be
14880, and 5700; the lesser of which (the errors
being alike, *i. e.* both too little) being taken from
the greater, there will remain 9180 for a dividend;
which divided by 72, the difference of the two er-
rors, quotes $127 \frac{1}{2}$, the true number of crowns *A.*
took up; as will appear, by adding 15 to them for
B's share, and dividing the sum of *A's* and *B's* by 5,

F

for

for C's share ; for so their particular numbers will be found to be as follow :

$$\begin{array}{r} A's \text{---} 127 \frac{1}{2} \\ B's \text{---} 142 \frac{1}{2} \\ C's \text{---} 54 \\ \hline \end{array}$$

The total of which is 324, the number of crowns given.

Or by the other rule ;

As 72, the difference of the errors (because alike) to 30, the difference of the suppositions :

$$\text{So is } \left\{ \begin{array}{l} 186 \\ \text{or} \\ 114 \end{array} \right\} \text{ to } \left\{ \begin{array}{l} 77 \frac{1}{2} \\ \text{or} \\ 47 \frac{1}{2} \end{array} \right\} \text{ which added } \left\{ \begin{array}{l} 50 \\ \text{or} \\ 80 \end{array} \right\} \text{ gives } 127 \frac{1}{2}, \text{ as before.}$$

EXAMPLE 2.

When first the marriage-knot was ty'd
Betwixt my wife and me,
My age did her's as far exceed,
As three times three does three.
But after ten and half ten years,
We man and wife had been,
Her age came up as near to mine,
As eight is to sixteen.

————— Now, pray,
What were our ages on the wedding-day ?

First, suppose her 21 then he must be 63
then adding to each 15 ————— 15
her age becomes 36 his 78

This error therefore is 3 years too few.

But supposing her 9, the error will prove 3 too many.

Setting down therefore against the suppositions the errors, with their respective marks, they will stand thus :

Sup-

[III]

Suppositions.

Errors.

$$\begin{array}{r} 21 \\ 9 \\ + \\ 3 \end{array}$$

Then multiplying cross-ways, 3 times 21 is 63
3 times 9 is 27

These products (because unlike) added, make 90 for a dividend, which divided by 6, the sum of the errors, quotes 15, her true age; prov'd thus:

$$\begin{array}{r} \text{She being } 15, \text{ he must be } \underline{\hspace{2cm}} 45 \\ 15 \text{ years added to each } \underline{\hspace{2cm}} 15 \end{array}$$

$$\text{Make her } 30, \text{ just half of his } \underline{\hspace{2cm}} 60$$

Or, by the other rule:

As 6, the sum of the errors, (because unlike) to 12, the difference of the suppositions:

So is the common error, 3 to 6; which either subtracted from 21, or added to 9, gives 15, (as before) for her age.

More **EXAMPLES.**

What number is that, to which if you add $\frac{1}{4}$ of itself, and from the sum subtract $\frac{1}{3}$ of itself, the remainder will be 210?

Answer, 229.

A gentleman caught a fish, whose head was 6 inches long, the tail as long as the head, and half as long as the body; the body was just the length of the head and tail. I demand the length of the fish?

Answer, 48 inches, or 4 foot.

C H A P. XI.

Of E X C H A N G E.

EXCHANGE is the receiving money in one country, for the value paid in another.

N O T E.

The Par of exchange is certain and fix'd, being always *par pro pari, like for like*, according to the weight and fineness of the coin.

But the course or current running price betwixt any two countries, rises and falls upon every occasion.

It would be both needless and endless to write of every kind of exchange; I shall therefore only give examples of the exchanges of *England*, with some other chief countries of *Europe*. And first, with

F R A N C E.

At *Paris, Lyons, Rouen, &c.* they keep their accounts in livres, sols, and deniers; and exchange upon the crown, the par of which, in sterling money is 4s. 6d.

N O T E.

NOTE.

12 Deniers }
 20 Sols } make { 1 Sol.
 3 Livres } { 1 Livre.
 { 1 Crown.

French money is changed into *Sterling* by this proportion;

As a Crown to the given rate :

So is the given *French* sum to the *Sterling* sought.

Or,

By supposing the given *French* crowns, goods, and the rate of exchange, the price; so casting up their value by *practice*.

EXAMPLES.

1st. What *sterling* money must be paid in *London*, to receive in *Paris* 438 crowns; exchange at 56 *d.* per Crown?

Cr.	d.	Cr.	Cr.	d.	s. d.
If 1	56	438	438 at 56 or 48 per Cr.		
		56			
		2628	4 s. $\frac{1}{5}$	87	12
		2190	8 d. $\frac{1}{6}$	14	12
			102	4	
		12)24528(			
		2 0(204 4			
		102 l. 4 s.			

2d. Change 537 crowns, 14 sols, 10 deniers, into Sterling money; exchange at 52 d. $\frac{1}{2}$ per crown.

Cr.	d.	Cr.	Sols.	De.	Cr.	Sols.	De.	d.
If 1—	52 $\frac{1}{2}$ —	537	14	10	537	14	10	at 52 $\frac{1}{2}$
60	2	60						
12—					4 s.	$\frac{1}{5}$	107	. 8
—105	32234				4 d.	$\frac{1}{12}$	. 8	19
720	12				$\frac{1}{2}$	$\frac{1}{8}$	. 1	2 : 4 $\frac{1}{2}$
					Sols.	$\frac{1}{6}$	 8	$\frac{3}{4}$
		3868	18		2	$\frac{1}{5}$	 1	$\frac{3}{4}$
		105			2	$\frac{1}{5}$	 1	$\frac{3}{4}$
				Deniers.	10	$\frac{1}{12}$	of 10 Sols.	$\frac{1}{2}$
		1934090						
		3868180						117 l. 10 s. 5 d. $\frac{1}{4}$ fac.
		2						

72|04061589|0(56410
 360
 ——— 12)28205(5
 461
 432 2|0)235|0
 ———
 295 117 l. 10 s. 5 d. $\frac{1}{4}$ facit.
 288
 ———
 78
 72
 ———
 69
 2
 ———
 72)138(1
 72
 ———
 66

Sterling money is changed into *French* by this proportion:

As the rate of exchange, to one crown :

So is the given *English* sum, to the *French* sought.

For E X A M P L E S.

Take the reverse of the former questions ; which will also be their proof.

1st. How many *French* crowns must be paid in *Paris*, to receive in *London* 102 *l.* 4 *s.* *Sterling* ; exchange at 56 *d.* per crown ?

d.	Cr.	l. s. d.
If 56	— 1 —	102 04
		20
		—
		2044
		12
		—
		Crowns,
		56)24528(438
		224
		—
		212
		168
		—
		448
		448
		—
		00
		00

2d. Change 117 l. 10 s. 5 d. $\frac{1}{4}$ Sterling into French crowns; exchange at 52 d. $\frac{1}{2}$ per crown.

d.	Crown.	l.	s.	d.
If 52 $\frac{1}{2}$	1	117	10	5 $\frac{1}{4}$
4		20		
210		2350		12
		28205		4
51				Crowns. Sols. Deniers.
60		21 0 11282 1(537	14	10
Sols,		105		
21 0 306 0(14		78		
21		63		
96		152		
84		147		
12		51		
12				
144				
66 the remainder of the other sum.				
21)210(10 deniers.				
21				
00				

More EXAMPLES.

If I pay in Paris 8297 crowns, 13 sols, 8 deniers, what may I draw my bill for to London, when the exchange is at 54 d. $\frac{1}{4}$ per crown?

Answer, 1875 l. 10 s. 4 d. $\frac{1}{2}$.

What Sterling money is equal to 954 crowns, 52 sols, when the exchange is at 58 d. $\frac{1}{8}$ per crown?

Answer, 231 l. 5 s. 1 d. $\frac{1}{2}$.

[117]

If I pay in *London* 536 *l.* what *French* money shall I receive in *Paris*; exchange at 57 *d.* per crown?

Answer, 2256 crowns, 50 sols, 6 deniers.

What *French* money is equal to 1529 *l.* 10 *s.* exchange at 53 *d.* $\frac{1}{4}$ per crown?

Answer, 6893 crowns, 31 sols, 3 deniers.

Secondly, with ITALY.

IN *Genoa*, *Leghorn*, &c. they keep their accompts in livres, sols, and deniers; and exchange upon the piece of eight, or dollar; the *par* of which with *London*, is also 4 *s.* 6 *d.* *Sterling*.

NOTE.

At *Genoa* 5 livres make a piece of eight, at *Leghorn* 6. Observe the rules for exchanging with *France*.

EXAMPLES.

How much *Sterling* shall I receive in *London*, if I pay in *Genoa* 820 dollars; exchange at 51 *d.* per dollar, or piece of eight.

Dollar.	d.	Dollars.	Dollars.
If 1	51	820	820 at 51 <i>d.</i> per dollar.
		51	4
		820	d. 3280
		4100	3 $\frac{1}{4}$ 205
		2)41820(	348 5
		2 0)348 5	174 <i>l.</i> 5 <i>s.</i>
		174 <i>l.</i> 5 <i>s.</i>	
		<i>£</i> 5	<i>Or,</i>

Or back again ;

If in *London* I pay 174 *l.* 5 *s.* *Sterling*, how many dollars shall I receive in *Genoa*; exchange at 51 *d.* per dollar?

d. *Dol.* *l.* *s.*
If 51 ——— 1 ——— 174 5
20

3485

12

51)41820(820

408

102

102

0

Venice exchanges by the ducat ; the *par* about 56 *d.* $\frac{1}{4}$ *Sterling*.

N O T E.

6 Solidi } make { 1 Gros.
24 Groffes } { 1 Ducat.

E X A M P L E.

Change 512 ducats into *Sterling* money ; exchange at 55 *d.* $\frac{1}{2}$ per ducat.

D. *d.* *D.*
If 1 ——— 55 $\frac{1}{2}$ ——— 512 *
2 512
—————
111

2)56832(

12)28416(

2)0 236|8

118 *l.* 8 *s.*

Ducats *d.*

512 at 55 $\frac{1}{2}$ per Duc.

4

d. 2048

6 $\frac{1}{2}$ 256

1 $\frac{1}{2}$ $\frac{1}{4}$ 64

236|8

118 *l.* 8 *s.*

* See Abbreviations of Multiplication.

Thirdly,

Thirdly, with SPAIN.

IN *Madrid, Seville, &c.* they keep their accompts in rials and mervadies, and exchange also by the piece of eight. The *par* with *London* 4 s. 6. d.

NOTE.

34 Mervadies } make { 1 Rial.
8 Rials } 1 Piece of $\frac{8}{8}$.

This exchange is also cast up by the foregoing rules.

EXAMPLE.

What *Sterling* money may I draw for to *London*, if I pay in *Seville* 4798 rials, 25 mervadies; exchange at 58 d. *Sterling* per piece of $\frac{8}{8}$?

Rials.

[120]

<i>Rials.</i>	<i>d.</i>	<i>Rials.</i>	<i>Merc.</i>
178	58	4798	25
34		34	

272

19197
14396

163157
58

1305256
815785
12

272)9463106(34790(2
816

210)28919(

1303
1088

144 19 2 3

2151
1904

2470
2448

226
4

272)904(3
816

88

Or

[121]

Or, back again.

Change 144*l.* 19*s.* 2*d.* $\frac{3}{4}$ Sterling into Spanish money.
Exchange at 58 *d.* per piece of $\frac{8}{8}$.

<i>d.</i>	<i>Rials.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>
If 58	8	144	19	2 $\frac{3}{4}$
4		20		

232

2899

12

34790

4

139163

8

232)11113304(4798 *Rials.*

928

1853

1624

2290

2088

2024

1856

168

34

672

504

88 Rem. of the other.

232)5800(25 *Mervadies.*

464

1160

1160

... 0

More

More E X A M P L E S.

Change 83927 *Rials* into pounds *Sterling*; exchange at 54 *d.* $\frac{1}{2}$ per piece of eight.

Answer, 2382 *l.* 6 *s.* 0 *d.* $\frac{1}{2}$

If I pay in *London* 5000 *l.* what *Spanish* money may I draw my bill for to *Madrid*; exchange at 57 *d.* $\frac{1}{8}$ per piece of eight?

Answer, 168052 *Rials*, 17 *Mervadies*.

Fourthly, with PORTUGAL.

IN *Lisbon*, *Oporto*, &c. they keep their accompts in *Rees*, and exchange on the *Mill-ree*; the *par* of which is about 6 *s.* 8 *d.* $\frac{1}{2}$ *Sterling*.

N O T E.

1000 *Rees* make 1 *Mill-ree*.

To make this exchange, still observe the same rule.

E X A M P L E S.

Change 48009 *rees* into *Sterling* money; exchange at 6 *s.* 5 *d.* per *mill-ree*.

<i>MR.</i>	<i>s.</i>	<i>d.</i>	<i>Rees.</i>
If 1	6	5	48009
1000	12		77

77

336063

336063

12

1|000)3696|693(3696)

4

2|0)30|8

2|772

15 *l.* 8 *s.* 0 *d.* $\frac{1}{2}$

Or,

Or, back again ;

Change 15 l. 8 s. 0 d. $\frac{1}{2}$ Sterling into rees ; exchange at 6 s. 5 d. per mill-ree.

	Rees.	l.	s.	d.
If 6 s. 5 d. ———	1000 ———	15	8	0 $\frac{1}{2}$
12		20		
77		308		
4		12		
308		3696		
		4		
		14786		
		1000		
	Rees.			

Note, 772, Rem. of 308) 14786772 (48009
the other stating, is 1232
here taken in.

2466
2464
2772
2772
0000

More EXAMPLES.

What Sterling money is equal to 595677 rees ; exchange at 6 s. 7 d. per mill-ree ?

Answer, 196 l. 1 s. 6 d. $\frac{1}{4}$.

Change 59 l. 12 s. into rees ; exchange at 6 s. 10 d. $\frac{1}{2}$ per mill-ree.

Answer, 173381 rees.

Fifthly,

Fifthly, HOLLAND, FLANDERS,
and GERMANY.

I Take these places together, because their accompts are kept, and their exchange with *England* made the same way.

For not only in *Amsterdam*, but also in *Antwerp* and *Hamburgh*, they keep their accompts in pounds, shillings, and pence *Flemish*; or guilders, stivers, and pennicks; and exchange with us upon our pound, giving us for it, when at *par*, 33s. 4d. *Flemish*.

N O T E.

Their pounds, shillings, and pence, are divided as ours; viz. their pound into 20 shillings, and their shilling into 12 pence.

Their other money is thus divided.

16 Pennicks } make { 1 Stiver.
20 Stivers } make { 1 Guilder.

Note also,

6 Stivers } make { 1 Shilling } *Flemish*.
6 Guilders } make { 1 Pound. }

Sterling money is changed into *Flemish* by this proportion; viz.

As 1 *Sterling* to the given rate:

So is the given *Sterling* to the *Flemish* sought.

Or,

By supposing the given *Sterling*, goods, and the given rate, their price, casting up their value by practice.

E X A M P L E

E X A M P L E.

If I pay in *London* 492 *l.* Sterling, what may I draw my bill for to *Amsterdam*; exchange at 34 *s.* 5 *d.* *Flemish* per pound Sterling?

<i>l.</i>	<i>s.</i> <i>d.</i>	<i>l.</i>
If 1 ———	34 5 ———	492
	12	413
Or by Practice.	413	1476
<i>l.</i>	<i>s.</i> <i>d.</i>	492
492 at 34 5 per <i>l.</i>		1968
<i>s.</i> 10 $\frac{1}{2}$ 246		
4 $\frac{1}{3}$ 98 8		12)203196(
4 $\frac{1}{12}$ 8 4		
1 $\frac{1}{4}$ 2 1		2 0)1693 3(
846 <i>l.</i> 13 <i>s.</i> facit:		846 <i>l.</i> 13 <i>s.</i> <i>Fl.</i>

If you would have the answer in guilders and stivers,
Divide the *Flemish* pence by 40, (the number of pence which make a *guilder*) and the quotient will be guilders, and if any thing remains, the half of it will be stivers, two *Flemish* pence being one stiver.

Or,

Multiply the *Flemish* pounds and shillings by 6, and it will produce guilders and stivers; for 6 guilders make 1 pound *Flemish*; and 6 stivers 1 shilling *Flemish*; and if there be any odd pence, multiply them by 8, for *Hennicks*.

E X A M P L E.

846 <i>l.</i> 13 <i>s.</i> the facit of the last sum.	
6	<i>Flemish</i> pence.
	4 0)20319 6(36 Rem.
5079 <i>G.</i> 18 <i>S.</i>	
	5079 <i>Guild.</i> 18 <i>Stivers.</i>
	<i>Flemish</i>

Flemish money (whether pounds, shillings, &c. or guilders, stivers, and pennicks) is exchanged into Sterling thus :

As the given rate to one pound Sterling ;
So is the given *Flemish* to the Sterling sought.

EXAMPLES.

Change 846 *l.* 13 *s.* *Flemish* into Sterling money, exchange at 34 *s.* 5 *d.* *Flemish* per *l.* Sterling.

<i>s.</i>	<i>d.</i>	<i>Flem.</i>	<i>l.</i>	<i>Sterl.</i>	<i>l.</i>	<i>s.</i>	<i>Flem.</i>
If	34	5	—	1	—	846	13
		12					20
		413				16933	
							12

413) 203196 (492 *l.* Sterling.
1652

3799

3717

826

826

00

Or,

Change 5079 guilders, 18 stivers, into Sterling money ; exchange 34 *s.* 5 *d.* per *l.* Sterling.

If

	<i>s. d. Flem.</i>	<i>l. Sterl.</i>	<i>Guild. Stiv.</i>
If	34 5	1	5079 18
	12		20

413

101598

2

413)203196(492 *l. Sterl.*

1652

3799

3717

826

826

0

More EXAMPLES.

Change 475 *l. Sterling* into *Flemish* money exchange ;
at 33 *s. Flemish* per *l. Sterling*.

Answer, 783 *l. 15 s. Flemish*.

What *Flemish* money is equal to 365 *l. 12 s.* exchange at 36 *s. 5 d.* per *l. Sterling* ?

Answer 665 *l. 13 s. 11 d. Flemish*.

If I pay at *Amsterdam* 5389 *l. 14 s. Flemish*, what *Sterling* money shall I receive at *London* ; exchange at 32 *s. 8 d. Flemish* per *l. Sterling* ?

Answer, 3299 *l. 16 s. 3 d. 3/4*.

Change 572 *l. 13 s. Sterling* into *guilders, stivers, &c.* exchange at 34 *s. 10 d. Flemish* per *l. Sterling*.

Answer, 5984 *guilders*, 3 *stivers*, 13 *pennicks*.

What may I draw my bill for to *London*, if I pay in *Antwerp* 7256 *guilders* ; exchange at 35 *s. 2 d. 1/2* per *l. Sterling* ?

Answer, 686 *l. 19 s. 2 d. Sterling*.

An

An Explanation of some Characters, sometimes used for Brevity's sake.

+ Is the mark of *Addition*, and shews the numbers it is placed between, are to be added together.

— Is the mark of *Subtraction*, and shews the latter of the numbers it stands between, is to be taken from the former.

$\times$ Is the mark of *Multiplication*, and shews the numbers on each side are to be multiply'd together.

$\div$ Is the mark of *Division*, and placed between two numbers, shews the former is to be divided by the latter. Or if one or several numbers are plac'd, part above a line, and part below it, it shews the upper are to be divided by the lower, and properly stands for the quotient.

$=$ Is the mark of *Equality*, and shews the number or numbers preceding it, are equal to that which follows it.

$\sqrt{}$ placed before any figures shews the Square Root of them is meant. And

$\sqrt[3]{}$ c before a number, denotes its Cube-Root.

$::$ Is the sign of *Proportion*, and being plac'd betwixt the two middle terms of 4 proportionals, signifies, *so is*, and $:$ is generally put between the first and second, and also between the third and fourth.

$\therefore$ is the mark for *therefore*.

To make all plain the Characters are thus us'd :

$$2 : 4 :: 6 : 12 \quad \sqrt[3]{4096} = 16, \quad \sqrt{16} = 4, \quad 4 + 2 = 6, \\ 6 - 2 = 4 \quad 4 \times 2 = 8, \quad \therefore 8 \div 2 = 4.$$

And thus read :

As 2 is to 4, so is 6 to 12 ; the Cube-Root of 4096 is, or is equal to 16 ; the Square-Root of 16 is equal to 4 ; 4 added to, or more by, 2, is equal to 6 ; and 6 made less by 2 is equal to 4 ; again, 4 multiplied by 2 is equal to 8 ; therefore 8 divided by 2 is equal to 4.

C H A P.

C H A P. XII.

Of Arithmetical and Geometrical PROPORTION or PROGRESSION.

PROGRESSION being a rule more curious than useful, and requiring more words for its full explication, than are consistent with this *Compendium*, I shall only give some (the most necessary) propositions in both its parts, arithmetical and geometrical; and refer the more inquisitive reader to Mr. Oughtred's *Clavis Mathematicæ*, for his further Satisfaction.

In all continued Proportion, or comparison of numbers, that number which is compared to another is called the *antecedent*, and the number to which it is compared is called the *consequent*.

There are two ways of comparing numbers with one another: 1st. with respect only to their simple difference, *i. e.* how much the one, *viz.* the *antecedent*, is greater, or less than the other, *viz.* the *consequent*, found by subtraction: thus the difference betwixt 5 and 8, is 3; which difference is called the *Arithmetical Ratio*. 2^d. Numbers are compared with one another, when we consider how often one number contains, or is contained in another; and this is found by division, the quotient being the *geometrical ratio*. Thus, 2 is the *geometrical ratio* betwixt 8 and 16.

ARITHMETICAL PROGRESSION.

ARITHMETICAL PROGRESSION is the regular increase or decrease of any rank of numbers by the continual addition or subtraction of some equal number.

So 1, 4, 7, 10, 13, 16, are in *Arithmetical Progression* to each other, equally increasing by the continual addition of 3.

And so also are 84, 82, 80, 78, regularly decreasing by the continual subtraction of 2.

Some of the most considerable properties of numbers in *Arithmetical Progression* are as follow :

1. That in any rank of numbers in *Arithmetical Progression* consisting of three, or any odd number of terms, the double of the middle term will be equal to the sum of the two extremes, or of any two mean, equally distant from the said middle term.

1, 2, 3, 4, (5), 6, 7, 8, 9.

Thus in the rank above 5, the middle term doubled is 10, = to $1+9$, the two extremes, or $2+8$, or $3+7$, or $4+6$, the several correspondent means.

2. That in any rank of numbers in *Arithmetical Progression* consisting of four, or any even number of terms, the sum of the two extremes will be equal to the sum of the two middle numbers, or of any two means equally distant from the said extremes,

2, 4, 6, 8, 10, 12, 14, 16.

Thus in the rank above $2+16$, the two extremes make 18, = $8+10$, the two middle numbers, or $6+12$, or $4+14$, the other correspondent means.

In PROGRESSION five things are to be noted.

V I Z.

- I. The first Term.
- II. The last Term.
- III. The number of Terms.
- IV. The equal Difference.
- V. The sum of all the Terms.

Any three of which being given, the other two may be found ; as Mr. *Oughtred* in his *Clavis Mathematicæ*, Chap. 19. Prop. the 6th, exemplifies in 20 propositions ; some of the most useful of which are as follow.

PROPOSITION I.

The last term, number of terms, and equal difference being given, to find the first term :

Or, the second, third, and fourth given, to find the first.

RULE.

Multiply the fourth by the third, made less by one, and subtract the product from the second, the remainder is the answer.

EXAMPLE.

A man takes out of his pocket, at 6 several times, 6 several numbers of shillings, every one exceeding the former by 6 ; the last was 42, what was the first ?

Answer, 12 s.

$$6 \times 5 = 30, 42 - 30 = 12 \text{ Answer.}$$

P R O.

PROPOSITION II.

The first, third, and fourth given, to find the second.

RULE.

From the product of the fourth, multiplied by the third, subtract the fourth; the remainder added to the first, gives the second.

EXAMPLE.

What's the last number of an *Arithmetical Progression* beginning at 4, and continuing, by the increase of 12 to 18 places? *Answer*, 208.

$$12 \times 18 = 216, 216 - 12 = 204, 204 + 4 = 208 \text{ Answer.}$$

PROPOSITION III.

The first, second, and fourth given, to find the third.

RULE.

Subtract the first from the second, and divide the remainder by the fourth; the quotient, with one added, gives the third.

EXAMPLE.

A traveller went 12 miles the first day, and increas'd every day's journey by 4 miles, till at last he went 64 miles in one day: How many days did he travel? *Answer*, 14 days.

$$64 - 12 = 52, 52 \div 4 = 13, 13 + 1 = 14 \text{ Answer.}$$

PROPOSITION IV.

The first, second, and third given, to find the fourth.

RULE.

R U L E.

Subtract the first from the second, and divide the remainder by the third, made less by one, the quotient gives the fourth.

E X A M P L E.

The ages of seven children increase by *Arithmetical Progression*; the youngest is two years old, the eldest 32: what's the common difference of their ages?
Answer 5 years.

$$32 - 2 = 30, 30 \div 7 - 1 = 5 \text{ Answer.}$$

P R O P O S I T I O N V.

The first, second, and third given, to find the fifth.

R U L E.

Half the sum of the first and second, multiply'd by the third, produces the fifth.

E X A M P L E.

12 persons give their charity to a poor man in *Arithmetical Progression*; the first gave 2 pence, the last 2 shillings, or 24 pence: how much did the poor man get? *Answer*, 156 pence, or 13 shillings.

$$2 + 24 = 26, 26 \div 2 = 13, 13 \times 12 = 156 \text{ Answer.}$$

G E O M E T R I C A L P R O G R E S S I O N.

GEOMETRICAL PROGRESSION is the increasing or decreasing of any rank of numbers by some equal *ratio*, that is, by the continual multiplication or division of some equal number.

G

Thus,

Thus, 2, 4, 8, 16, 32, increase in *Geometrical Progression*, by a double *ratio*, or continual multiplication by 2; and 405, 135, 45, 15, 5, decrease after the same manner by a continual division by 3, or by a triple *ratio*.

Some of the most considerable properties of numbers in *Geometrical Proportion* are as follow:

1. That in any rank of numbers in *Geometrical Progression*, consisting of three, or any odd number of terms, the product of the middle term, multiply'd by itself, will be equal to the product of the two extremes, or of any two means equally distant from the said middle term.

1, 2, 4, (8), 16, 32, 64.

Thus, in the rank above, 8, the middle term squar'd, *i. e.* multiply'd by itself, is $64 = 1 \times 64$, the two extremes, or 2×32 , or 4×16 , the correspondent means.

2. That in any rank of numbers in *Geometrical Progression*, consisting of four, or any even number of terms, the product of the two extremes will be equal to the product of the two middle numbers, or of any two means equally distant from the said extremes.

2, 4, 8, 16, 32, 64.

Thus in the rank above 2×64 , the two extremes produce $128 = 8 \times 16$, the two middle numbers, or to 4×32 , the other correspondent means.

In Geometrical Progression the same five things are to be noted as in Arithmetical Progression:

V I Z.

- I. The first Term.
- II. The last Term.
- III. The number of Terms.
- IV. The *ratio*, or equal difference.
- V. The sum of all the Terms.

Note,

Note, Figures are sometimes set over a rank of geometrical proportionals, as here-under, and are called *Indices* or *Exponents*; serving to shew the distance of any term from unity, or from the first term of the series

Thus

0	1	2	3	4	5	6
1	2	4	8	16	32	64

PROPOSITION I.

To find any remote term of a *Geometrical Progression*, proceeding from unity, whose *ratio* or common difference is known, without producing all the intermediate terms.

R U L E.

Find some few of the leading terms, and place over them their exponents; then multiplying the last-found term by itself, it will produce a term double thereto; which again multiplied by itself, will produce another double to the last: Thus proceed, till either you produce the term sought, or one a little short of it; which may be completed by multiplying it again by that term, which stands under such exponent as would make up the number.

Or, in other words, observe what figures of the exponents found added together, would give the exponent of the term wanted, and the numbers standing under the said exponents, multiply'd into each other, will produce the term requir'd.

From whence may be observed the coherence or agreement betwixt numbers in arithmetical and geometrical proportion.

E X A M P L E.

One agrees for 14 oranges, to pay only the price of the last, at a farthing for the first, an half-penny the second, &c. still doubling the price for the next, what must he give?

G 2

Here

Here, setting down only the five first terms, with their exponents, the fourteenth may thus be found, $5+5+4=14$. So the terms under those exponents being multiplied into each other, give the term answering. But as the exponent 1 stands always over the second term, so the exponent 13 answers to the 14th term; therefore add only 3 to the two 5's; and so always that exponent must be taken, which is one short of the term requir'd; as for 20, 19; for 30, 29, &c. which must be carefully noted.

See the work of the foregoing example.

0 . 1 . 2 . 3 . 4 . 5	
1 . 2 . 4 . 8 . 16 . 32	
<u>Exponents.</u>	<u>Terms.</u>
5	32 the 5th multiply'd by
5	32 itself.
3	—
—	64
13	96
	—
	gives 1024 the 10th, which
	multiplied by —8 the 3d Term,
	—
	gives 8192 the 14th.

And so many farthings, or 8*l.* 10*s.* 8*d.* must he give at the rate agreed.

PROPOSITION II.

To find any remote term of a *Geometrical Progression*, not proceeding from unity, whose *ratio* and first term is given, without producing all the intermediate terms.

RULE.

R U L E.

Set down, as before, some few of the leading terms with their exponents, and multiply also (as in the first proposition) the terms under such exponents, as added, would make the exponent short by one of the terms required; only remember, that every product must here be divided by the first term.

E X A M P L E.

A sum of money is thus to be divided among 9 persons, the 1st to have 50*l.* the 2d 150*l.* and so on in that proportion, one three times more than the other, to the last; what will the last have?

Setting down the five first terms with their exponents, thus,

$$\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 \\ 50 & 150 & 450 & 1350 & 4050 \end{array}$$

Multiply the last term by itself, and divide the product by 50, the first term, the quotient will be 328050 the 9th term required, answering to the 8th exponent.

Or setting down but the three first terms, multiply the last by itself, and divide the product by the 1st term, it quotes the 5th; which again multiplied by itself, and divided by the first, will quote the 9th, as before. See the work.

$$\begin{array}{r} \begin{array}{cccc} 0 & 1 & 2 & \end{array} \\ \begin{array}{cccc} 50 & 150 & 450 & \end{array} \end{array} \quad \begin{array}{r} \begin{array}{cc} 450 & 4050 \\ 450 & 4050 \end{array} \\ \hline \begin{array}{cc} 22500 & 202500 \\ 1800 & 162000 \end{array} \\ \hline \begin{array}{cc} 5|0)20250|0(& 5|0)164250|0(\\ \hline 4050 & 328050 \end{array} \end{array}$$

Answer, 328050*l.* for the 9th person.

P R O P O S I T I O N III.

Having the first term, the common excess, and the number of places given, to find the total sum of the whole progression.

R U L E.

Having found the last term by the foregoing rules, subtract from it the first, and divide the remainder by the common excess, made less by one; the quotient is the sum of all, except the last; to which that being also added, gives the total required.

Note. If the *ratio* be double, that is, if the common excess be 2, the difference between the least and the greatest term, added to the greatest, gives the total sum.

If the *ratio* be triple, $\frac{1}{2}$ the difference must be added; if quadruple, $\frac{1}{3}$, &c.

E X A M P L E S.

1st. A coffee-man, upon the signing of the last peace, for 50 guineas down, agreed to pay the first day one coffee-berry, the second two, the third four, and so on, to double the quantity every day, till the same was proclaimed. What number of berries would it amount to, supposing the time 60 days; and what would their value be, supposing 1000 berries to the pound, and the pound to be sold at 5 s?

See the work.

Days,

1 — 1 Days. 16 Berries.

2 — 2 16

3 — 4 96

4 — 8 16

5 — 16 256

2

512 the 10th day.

16

3072

512

8192

2

16384 the 15th day.

16384

65536

131072

49152

98304

16384

268435456

2

536870912 the 30th day.

536870912

1073741824

536870912

4831838208

37580963840

4294967296

3221225472

1610612736

2684354560

288230376151711744

2

576460752303423488 the 60th day.

1|000(1152921504606846|975 Tot. number of berries.

5s. $\frac{1}{4}$ 288230376151711 l. 10s. their val. at 5s. per lb.

Ex. 2. A Grazier offers 40 oxen for a farthing a head, and treble it throughout; to what sum will it amount?

See the work.

Oxen. Farthings.

1	1	1162261467
2	3	1162261467
3	9	
4	27	8135830269
5	81	6973568802
	81	4649045868
		1162261467
	81	6973568802
648		2324522934
		2324522934
6561		6973568802
	3	1162261467
		1162261467
19683	10th Ox.	
19683		1350851717672992089
		3
59049		
157464		4052555153018976267 40th Ox.
118098		
177147		6078832729528464400 Tot. Far.
19683		Or,
		63321174265921501. 8s. 4d.
387420489		
	3	
1162261467	the 20th Ox.	

C H A P. VIII.

Of VULGAR FRACTIONS.

A FRACTION is a part or parts of an unit, and written with two figures, with a line drawn between them, as $\frac{3}{4}$, $\frac{2}{5}$, $\frac{6}{8}$, each of which really stands for the quotient of a division made of the upper figure divided by the lower; so that if the upper figure can be multiply'd into any number which the unit is equal to in known parts, and that product divided by the lower figure, the quotient will give the number of those parts equal to the fraction.

$$\text{As } 1 \text{ l.} = 20 \text{ s.}, \quad \frac{3}{4} \text{ l.} = \frac{60}{4} \text{ s.} = 15 \text{ s.}$$

Or,

$$1 \text{ s.} = 12 \text{ d.}, \quad \frac{3}{4} \text{ s.} = \frac{36}{4} \text{ d.} = 9 \text{ d.}$$

The figure under the line is call'd the *denominator*, because it gives name to the fraction, and also shews into what parts the unit is broken; and that above, the *numerator*, because it tells how many of such parts are meant by the fraction. Thus the fraction $\frac{3}{4}$ shews, that the unit is divided into 4 parts, and that 3 of those parts are thereby express'd.

1. Therefore, if the *numerator* and *denominator* of a fraction should be equal, as $\frac{4}{4}$, $\frac{6}{6}$, the value of such fraction would be exactly an unit or integer; for, by the definition above, the *denominator* shews into how many parts the unit is broke; and the *numerator* expresses how many of those parts are meant by the fraction: So that, in the fraction $\frac{4}{4}$, the *denominator* declaring the unit to be broke into 4 parts, and the *numerator* expressing 4, that is, all of those parts, it is plain the said fraction is equal to an unit or whole number; because the sum of all the parts must be equal to the whole. From which observation it is also plain, that so often as the *denominator* is contain'd in the *numerator*, so many whole numbers or units are contain'd

in such a fraction. And this may serve as a reason for the operations of the 1st, 2d and 3d sorts of Reduction.

2. As all proper fractions may be suppos'd to arise from the remainders of divisions, when the divisor can no longer measure the dividend; so every fraction may be look'd upon as the two given terms of a division, the *numerator* as the dividend, and the *denominator* as the divisor, according to the latter part of the character, page 128, to signify division; from whence it appears, that if the *numerator* and *denominator* of a fraction be either multiply'd or divided both by the same number, the products or quotients will still remain in the same proportion, and the new fraction so arising be of the same value with that given. Thus the *numerator* and *denominator* of the fraction $\frac{2}{4}$, multiply'd by 2, will produce $\frac{4}{8}$; or, divided by 2, will quote $\frac{1}{2}$; all which fractions are of the same value; 4 bearing the same proportion to 8, and 1 to 2, as 2 does to 4. And from hence may appear the reason of the 5th and 6th sorts of Reduction.

Of Vulgar Fractions there are four sorts, viz.

1st. A *proper*; whose *numerator* is less than its *denominator*, as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{6}$, &c.

2d. An *improper*; whose *numerator* is equal to, or greater than, its *denominator*, as $\frac{5}{3}$, $\frac{6}{4}$, &c.

3d. A *compound*, or *fraction of a fraction*; known by the word *of*, as $\frac{3}{4}$ of $\frac{5}{7}$, &c.

4th. That which is join'd with a whole number, as $5\frac{1}{2}$, $3\frac{2}{3}$, &c. call'd a *mix'd number*.

Before fractions can be either added or subtracted, they must be reduc'd into one denomination, that is, each to have the same *denominator*; and therefore, before we proceed to those rules, we must learn Reduction.

REDUCTION

REDUCTION of FRACTIONS.

IN *Reduction of Fractions* there are seven varieties.

First, To reduce a whole number into an *improper fraction*; which is done by placing 1 for its *denominator*.

EXAMPLES.

Reduce 4 to a Fraction, *facit* $\frac{4}{1}$.

or 18, *facit* $\frac{18}{1}$.

But if you would assign it any other *denominator*, multiply the whole number by the *denominator* assign'd, and place the product for a *numerator* over the assign'd *denominator*.

EXAMPLES.

Reduce 12 to a fraction, whose *denominator* let be 8.

$\frac{8}{96}$ *facit* $\frac{96}{8}$.

Or if 6 were to be made a fraction, and its *denominator* to be 7, it would become $\frac{42}{7}$.

For the reason of this rule consider the foregoing remarks.

Second, To reduce a mix'd number into an improper one.

Multiply the integral parts of the number by the *denominator* of the fraction given, and take in the *numerator*; the product place for a new *numerator* over the *denominator* given.

E X A M P L E S.

Reduce $5\frac{1}{3}$ to an improper fraction, *facit* $\frac{16}{3}$.

$$\begin{array}{r} \text{---} \\ 16 \\ \text{---} \\ 3 \end{array}$$

So $12\frac{1}{2}$ reduc'd to an improper fraction, will be $\frac{25}{2}$.

The reason of this rule is the same as the foregoing, there being no difference in the operation, but the taking in the given *numerator*.

Third, To reduce an improper fraction into its equivalent whole or mix'd number.

R U L E.

Divide the *numerator* by the *denominator*, the quotient gives the whole number contain'd: But if any thing remains, (as in the 2d example) it must be plac'd as a new *numerator* over the given *denominator*.

E X A M P L E S.

Reduce $\frac{32}{8}$ to its equivalent whole number.

$$8)32(4 \text{ facit.}$$

Reduce $\frac{45}{6}$ $6)45(7\frac{3}{2}$ *facit.*

$$\begin{array}{r} \text{---} \\ 42 \\ \text{---} \\ 3 \end{array}$$

This being only the reverse of the foregoing rule, the same reason still holds.

Fourth,

Fourth, To reduce a compound fraction to a single one, *i. e.* a fraction of a fraction, to a fraction of an unit, which is the multiplication of them into one another.

R U L E.

Multiply all the *numerators* together for a *numerator*, and all the *denominators* for a *denominator*, the fraction thus form'd is the product.

E X A M P L E S.

Reduce $\frac{2}{3}$ of $\frac{1}{4}$ to a single fraction, *facit* $\frac{2}{12}$.

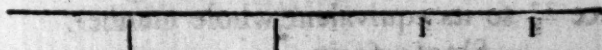
Reduce $\frac{4}{5}$ of $\frac{6}{8}$ of $\frac{7}{9}$ to a single fraction, *facit* $\frac{168}{360}$.

See the Work—

$\begin{array}{r} 4 \\ 6 \\ \hline 24 \\ 7 \\ \hline 168 \end{array}$	$\begin{array}{r} 5 \\ 8 \\ \hline 40 \\ 9 \\ \hline 360 \end{array}$
---	---

So also $\frac{3}{8}$ of $\frac{9}{10}$ is $\frac{27}{80}$.

The reason of this operation will best appear by representing the unit by a line, which, according to the first example, must be suppos'd to be divided into 4 parts, and each of those parts again into 3 smaller parts; thus,



Then, as $\frac{1}{4}$ of the unit will denote 1 space between the larger divisions, so $\frac{2}{3}$ of that fourth must signify only 2 spaces of the lesser divisions; consequently, if I would express what part of the whole line $\frac{2}{3}$ of $\frac{1}{4}$ is, it will plainly appear to be $\frac{2}{12}$: or, had the compound fraction been $\frac{2}{3}$ of $\frac{3}{4}$, the single fraction equal to it had been $\frac{6}{12} = \frac{1}{2}$. This shews, that multiplying a fraction or any quantity by a proper fraction decreases it in the same proportion as such proper fraction is less than an unit.

Fifth, To reduce a fraction into its lowest terms.

R U L E.

Divide the *numerator* and *denominator* by any figure; fo

so that nothing may remain, and the quotients will be a new fraction of the same value with that given.

The rule generally given for finding the greatest common measure, or number, to divide your given fraction by, is this :

Divide the *denominator* by the *numerator*, and the *numerator* by the *remainder*, if any : so continuing to make the last divisor the dividend, and the remainder the divisor, till nothing remains, the last divisor will be the greatest common measure, or number, by which you can divide your fraction.

E X A M P L E.

What is the greatest common measure by which $\frac{208}{684}$ can be divided ?

$$208)684(3$$

$$\underline{624}$$

$$\cdot 60)208(3$$

$$\underline{180}$$

$$\cdot 28)60(2$$

$$\underline{56}$$

$$\text{Answer, } 4)28(7$$

$$\underline{28}$$

$$0$$

$\frac{208}{684}$ then, being divided by 4, gives $\frac{52}{171}$, which are its lowest terms.

But this way of finding the common measure is too tedious, often making more work than it saves. Observe therefore these more practical directions :

If you cannot at once discover the greatest number you may divide by, if both your *numerator* and *denominator* are even numbers, you may always halve them, and often that way reduce your fraction to advantage.

Thus

Thus the fraction given in the foregoing example, divided twice by 2, gives, as before, $\frac{52}{171}$.

$$2 \frac{208}{684} \bigg| \frac{104}{342} \bigg| \frac{52}{171}$$

and $\frac{192}{576}$ may be reduced, by continual halving, to $\frac{3}{9}$.

$$2 \frac{192}{576} \bigg| \frac{96}{288} \bigg| \frac{48}{144} \bigg| \frac{24}{72} \bigg| \frac{12}{36} \bigg| \frac{6}{18} \bigg| \frac{3}{9}$$

Also, when both your *numerator* and *denominator* have cyphers on the right hand, you may abbreviate the fraction, by striking off an equal number from both.

$$\text{Thus } \frac{45}{96} \bigg| \frac{00}{00}, \text{ or } \frac{825}{960} \bigg| \frac{00}{00}.$$

Or, if the right hand figures of your *numerator* and *denominator* are both fives, or one a five, and the other a cypher, (as the remainder of the last example) you may always divide them by 5.

$$\text{Thus } \frac{825}{960} \text{ divided by } 5.$$

$$\text{is } \frac{165}{192}, \text{ equal to } \frac{825}{960}.$$

$$\text{So also is } \frac{125}{365} \text{ reduced to } \frac{25}{73}.$$

Thus, in the Rule of Three, we abbreviate our statements by the same rule, and on the same reason; for the second and third numbers multiply'd together, are as a *numerator* to the first number, their *denominator*.

For the reason of this rule, see the second remark.

Sixth, to reduce several fractions given with different denominators to have a common denominator.

R U L E.

Multiply each of the given *numerators* into all the given *denominators*, except its own; and the several products

products will be so many new *numerators*, whose common *denominator* must be the product of all the given *denominators* multiply'd together.

E X A M P L E S.

Reduce $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{6}{10}$ to have a common denominator.

To do which,

First, The *numerator* 3 must be multiply'd by the *denominators* 8 and 10, which will produce 240.

Secondly, the *numerator* 5, multiply'd by the *denominators* 4 and 10, will produce 200

Thirdly, The *numerator* 6, multiply'd by the *denominators* 4 and 8, will produce 192.

} *New numerators.*

Then the three *denominators* 4, 8, 10, multiply'd together, will give 320 for a common *denominator*.

See the work.

1 st Numb.	2 ^d Numb.	3 ^d Numb.	Denomin.
3	5	6	4
8	4	4	8
—	—	—	—
24	20	24	32
10	10	8	10
—	—	—	—
240	200	192	320
—	—	—	—
$\frac{240}{320}$	$\frac{200}{320}$	$\frac{192}{320}$	

So $\frac{5}{10}$, $\frac{3}{8}$, $\frac{4}{12}$, and $\frac{9}{15}$, reduced to have a common Denominator will be $\frac{7200}{14400}$, $\frac{5400}{14400}$, $\frac{4800}{14400}$, $\frac{8640}{14400}$

See the work.

1st Num.	2d Num.	3d Num.	4th Num.	Denom.
5	3	4	9	10
8	10	10	10	8
40	30	40	90	80
12	12	8	8	12
480	360	320	720	960
15	15	15	12	15
7200	5400	4800	8640	14400

The foregoing is the common Rule for reducing several fractions with different denominators, to have a common denominator, but as, by abbreviation fractions are often reduc'd into much lower terms, than they are at first given; so here, a common denominator may sometimes be found much smaller than that arising by the preceding method in the following manner.

R U L E.

Having first, by the fifth sort of Reduction, reduc'd all your given fractions to their lowest terms, cancel or strike out all such of the *denominators* as are *aliquot* parts of others; then drawing a line under all the *denominators*, if two or more can be divided by any number, so that nothing may remain, divide by such number; and place the quotients under the line, and with them, in their proper places, all such of the *denominators* as would not admit of such division. If any of the said quotients or *denominators* so brought down, are capable of further abbreviation, draw another line, and proceed as before, so continuing as low as possible; then multiply continually, one into another, the several divisors, and the quotients arising, (with the remaining denominators if any) under the last line, and their product will be the least common denominator.

The

The new *numerators* to which may be found, by dividing the said common *denominator* by each given *denominator*, and multiplying the quotients by their respective *numerators*.

E X A M P L E.

Reduce $\frac{3}{4}$ $\frac{5}{8}$ $\frac{6}{10}$ to have the least common denominator. 2 ———

Answer, $\frac{4}{40}$ $\frac{5}{40}$ $\frac{24}{40}$

The common *denominator* 40 is thus produced.

First, The *denominator* 4 being an *aliquot* part of 8, is cancelled, then the *denominators* 8 and 10 are divided by 2. *Lastly*, The divisor $4 \times 5 = 40$.

The reason of this sixth sort of reduction is evident from the 2d of the preceding remarks; for as both the *numerator* and *denominator* of each given fraction are equally multiply'd by all the other *denominators*, consequently the new fractions thence arising, must be equal to the fractions given.

Seventh, To value a fraction, or reduce it to the known parts of an integer.

R U L E.

Multiply the *numerator* by the next inferior denomination, and divide the product by the *denominator*; the quotient shews the parts sought, and the remainder becomes a new *numerator* to the given *denominator*; which must still be valued by the same rule, proceeding till you have brought it into the least known parts of the integer.

E X A M.

EXAMPLES.

1st, What's the value of $\frac{3}{5}$ of a pound sterling?

$$\begin{array}{r} 3 \\ 20 \\ \hline 5)60(\\ 12 \text{ Shillings.} \end{array}$$

2^d. What's the value of $\frac{13}{49}$ of a pound sterling?

$$\begin{array}{r} 13 \\ 20 \\ \hline \text{— s. d. qrs.} \\ 49)260(5 \quad 3 \quad 2 \quad \frac{34}{100} \\ \text{—} \\ 15 \\ 12 \\ \hline 49)180(3 \\ \text{—} \\ 33 \\ 4 \\ \hline 49)132(2 \\ \text{—} \\ 34 \end{array}$$

3^d. What hundreds, quarters, and pounds, are contained in $\frac{53}{94}$ of a ton?

$$\begin{array}{r} 53 \\ 20 \\ \hline \text{— C. gr. lb.} \\ 94)1060(11 \quad 1 \quad 2 \quad \frac{92}{100} \\ \text{—} \\ 120 \\ \text{—} \\ 26 \\ 4 \\ \hline 94)104(1 \\ \text{—} \\ 10 \\ 28 \\ \hline 94)280(2 \\ \text{—} \\ 92 \end{array}$$

There will be no difficulty in accounting for this rule, if we consider but the particular working of any one example: Thus in the first, $\frac{3}{5}$ of a pound sterling are given to be valued: Now as 20 s. make a pound, so consequently any part of a pound must be 20 times as great a part of a shilling: Therefore $\frac{3}{5}$ of a pound make $\frac{60}{5}$ of a shilling; which being an improper fraction

tion its *numerator* is divided by its *denominator*, to find the units or whole numbers (which in this case must be shillings) contain'd in it; according to the directions of the 3d sort of Reduction.

By this rule are remainders of statings in the Rule of Three, &c. valued. See page 25.

ADDITION of FRACTIONS.

WHEN the given fractions are parts of one common denominator, all you have to do is, to add the numerators together and place the sum for a new numerator over the given common denominator; thus $\frac{1}{6}$, $\frac{2}{6}$, and $\frac{3}{6}$ added together, - make $\frac{6}{6}$ equal to $1 \frac{2}{6}$.

But when amongst the given fractions there are either compound ones or single with different denominators, they must be prepared by reduction, before they can be added.

EXAMPLES.

1st. What's the sum of $\frac{3}{6}$, $\frac{2}{6}$, and $\frac{4}{7}$?

According to the sixth sort of reduction the given fractions being reduced to a common denominator they will be $\frac{189}{504}$, $\frac{112}{504}$, $\frac{288}{504}$, the numerators of which being added the sum is $\frac{589}{504}$ equal to $1 \frac{85}{504}$.

2^d. Add $\frac{3}{4}$ of $\frac{3}{8}$, and $\frac{9}{10}$ of $\frac{6}{7}$ and $\frac{5}{8}$ together.

Reduce the compound fractions to singles ones, by the fourth rule of reduction; and instead of $\frac{3}{4}$ of $\frac{3}{8}$, and $\frac{9}{10}$ of $\frac{6}{7}$, you will have $\frac{9}{32}$ and $\frac{54}{70}$, to add to your $\frac{5}{8}$; all which single fractions being reduced, first into their lowest terms, and then to a common denominator, they will be $\frac{315}{1120}$, $\frac{864}{1120}$ and $\frac{700}{1120}$; which added, make $\frac{1879}{1120}$, equal to $1 \frac{759}{1120}$.

3d. Add $4\frac{1}{2}$ and $3\frac{1}{6}$ and $2\frac{2}{7}$ together.

When mix'd numbers are given to be added, reduce only their fractional parts to one denominator, and add their sum to the total of the whole numbers.

$$\begin{array}{r}
 4\frac{2}{8} \quad 3\frac{2}{4} \quad 2\frac{2}{4} \\
 4 \quad \quad \quad 3 \quad \quad \quad 2 \\
 \hline
 42 \quad 14 \quad 24 \\
 \hline
 80 \\
 \text{facit } 7 \quad \hline
 84
 \end{array}$$

More **E X A M P L E S.**

Add $8\frac{1}{2}$ and $3\frac{1}{10}$ and $1\frac{7}{15}$ together.

Facit $13\frac{1}{30}$.

What's the sum of $\frac{1}{4}$ of $\frac{7}{8}$, and $4\frac{1}{2}$ and $1\frac{2}{3}$?

Answer, $5\frac{1}{24}$.

SUBTRACTION of FRACTIONS.

Subtraction of Fractions is also nothing (after the given fractions are prepared by reduction) but taking one numerator from the other.

E X A M P L E S.

1st. From $\frac{3}{4}$ take $\frac{2}{5}$

prepared $\frac{27}{36}$ $\frac{16}{36}$ $\frac{11}{36}$ Remainder.

zd. From $5 \frac{2}{3}$ take $\frac{9}{10}$.

First, The mix'd number $5 \frac{2}{3}$ being reduced to the improper fraction $\frac{17}{3}$, by the second rule of reduction proceed as before.

From $\frac{17}{3}$ take $\frac{9}{10}$, or prepar'd.

From $\frac{170}{30}$ take $\frac{27}{30}$ Remains $\frac{143}{30}$ or $4 \frac{23}{30}$.

Or without reducing the mix'd number to an improper fraction, make only the fractional parts of one denomination, and you will then have $\frac{27}{30}$ to take from $\frac{20}{30}$; which because you cannot do, you must borrow an unit, *i. e.* $\frac{30}{30}$; then, $\frac{27}{30}$ from $\frac{30}{30}$, there will remain $\frac{3}{30}$; to which adding the $\frac{20}{30}$, the remainder will then be $\frac{23}{30}$; and the unit borrowed being deducted from the whole number 5, there will remain in all $4 \frac{23}{30}$, as before.

MORE EXAMPLES.

From 5 l. $\frac{7}{8}$ take $\frac{9}{10}$ of $\frac{5}{6}$ Rem. 5 l. $\frac{1}{8}$.

From $\frac{19}{20}$ take $\frac{1}{7}$ of $\frac{2}{3}$ Rem. $\frac{359}{420}$.

MULTIPLICATION of FRACTIONS.

IF the numbers given are whole, or mixed, they must first be brought into improper fractions; or if they are compound fractions, they must be reduced to single ones, or taken as part of the fractions to be multiplied together; but if the numbers given are either proper or improper single fractions, you have only to multiply the two given *numerators* together for a new *numerator*, and the given *denominators* for a *denominator*. The reason of which is clear, from the explanation of the fourth sort of reduction, which is this rule itself.

E X A M.

EXAMPLES.

1st. Multiply $\frac{4}{8}$ by $\frac{3}{6}$, *facit* $\frac{12}{48}$, or by abbreviation $\frac{1}{4}$.

2d. Multiply $\frac{2}{6}$ by $\frac{4}{7}$, *facit* $\frac{8}{42}$, or $\frac{4}{21}$.

3d. Multiply 4 by $\frac{3}{8}$. 4 by the first sort of reduction, reduced to a fraction, is $\frac{4}{1}$; which multiply'd by $\frac{3}{8}$, makes $\frac{12}{8}$, or $1\frac{1}{2}$.

4th. Multiply $7\frac{1}{2}$ by $8\frac{2}{3}$. These mix'd numbers reduced by the 2d rule of reduction to improper fractions, make $\frac{15}{2}$ and $\frac{26}{3}$; which multiply'd together, make $\frac{390}{6}$, or 65.

It may here be required to shew how mix'd numbers may be multiplied together without reducing them to improper fractions, which in many cases may shorten the work.

Suppose it, as in the last example, required to multiply $7\frac{1}{2}$ by $8\frac{2}{3}$.

First, 8 times $\frac{1}{2}$ or ($\frac{1}{2}$ of 8) is 4, without any fraction remaining, which carried to (8 times 7) 56 makes 60 to be set down as in multiplication of integers; now to multiply by $\frac{2}{3}$, we may first take $\frac{1}{3}$ of $7\frac{1}{2}$, thus, $\frac{1}{3}$ of 7 is 2, and 1 remains, which makes $\frac{1}{2}$ up $\frac{3}{2}$, and $\frac{1}{3}$ of $\frac{3}{2}$ is $\frac{1}{2}$, so $2\frac{1}{2}$ twice set down, and added to the 60, make up the product 65, as before.

$$\begin{array}{r} 7\frac{1}{2} \\ 8\frac{2}{3} \\ \hline 60 \\ 2\frac{1}{2} \\ 2\frac{1}{2} \\ \hline 65 \end{array}$$

Again,

Again, Suppose it required to multiply $3\frac{3}{4}$ by $3\frac{3}{4}$.

First, 3 times $\frac{3}{4}$ is $(\frac{9}{4}) 2\frac{1}{4}$. I set down the $\frac{3}{4}$, and carry the 2 to (3 times 3) 9, which makes 11 to be set down as an integer; now $\frac{3}{4}$ of $3\frac{3}{4}$ may be taken as in the rule of Practice, first the half of it, and then the half of that half, as the $\frac{1}{2}$ of 3 is 1, and 1 remains, that makes the $\frac{3}{4}$ up $\frac{7}{4}$, the half of $\frac{7}{4}$ is $\frac{7}{8}$; now half of $1\frac{7}{8}$ is $\frac{15}{16}$. These fractions are easily reduced, and added up with the integers.

$$\begin{array}{r} 3\frac{3}{4} \\ 3\frac{3}{4} \\ \hline 11\frac{1}{4} \\ 1\frac{7}{8} \\ \hline 14\frac{15}{16} \end{array}$$

Let it be required to multiply $892\frac{3}{11}$ by $78\frac{7}{8}$

First, 8 times 892 is 7136, and 70 times 892 is 62440; then taking the $\frac{7}{8}$ of 892 by several parts as in Practice, thus, half 892 is 446, half of 446 is 223, and half of 223 is $111\frac{1}{2}$; now for the $\frac{3}{11}$ of $78\frac{7}{8}$, first 3 times $78\frac{7}{8}$ is $236\frac{5}{8}$, which divided by 11, quotes $21\frac{45}{88}$, and these added is the product, as shewn at the side.

$$\begin{array}{r} 892\frac{3}{11} \\ 78\frac{7}{8} \\ \hline 7136 \\ 62440 \\ 446 \\ 223 \\ \hline 111\frac{1}{2} \\ 21\frac{45}{88} \\ \hline 70378\frac{1}{88} \end{array}$$

MORE EXAMPLES.

Multiply $9\frac{1}{2}$ by $\frac{2}{3}$, *facit* $3\frac{1}{3}$.
Multiply $\frac{7}{8}$ by 8, the product is 7.

This last example helps to clear up the reason given for the rule in Division, by shewing that multiplying a fraction by its denominator takes it away, and makes the numerator a whole number.

DIVISION of FRACTIONS.

WHEN you have made the same preparation of your numbers as directed in Multiplication, multiply the numerator of your dividend by the denominator of your divisor, for a numerator; and the numerator of your divisor by the denominator of your dividend, for a denominator; the fraction thus form'd is the quotient.

EXAMPLES.

1st. Divide $\frac{7}{9}$ by $\frac{3}{8}$, *facit* $\frac{56}{27}$, or $2\frac{2}{27}$.

2^d. Divide $\frac{7}{8}$ by $\frac{1}{3}$, *facit* $\frac{21}{8}$, or $2\frac{5}{8}$.

3^d. Divide 7 by $\frac{3}{4}$, prepared by the first rule of Reduction, it will be $\frac{7}{4}$ by $\frac{3}{4}$; *facit* $\frac{28}{3}$, or $9\frac{1}{3}$.

4th. Divide $4\frac{1}{2}$ by $2\frac{3}{8}$, by the 2^d rule, it is $\frac{9}{2}$ by $\frac{19}{8}$, which divided gives $\frac{72}{19}$, or $3\frac{6}{19}$.

5th. Divide $\frac{2}{3}$ of $\frac{1}{9}$ by $\frac{4}{8}$ of $\frac{7}{8}$, reduced by the 4th Rule $\frac{2}{27}$ by $\frac{28}{64}$, *facit* $\frac{128}{729}$.

6th. Divide $9\frac{1}{2}$ by $\frac{2}{5}$ of $\frac{3}{4}$. *facit* $\frac{360}{12}$, or $31\frac{2}{3}$.

The reason for this rule is, that the numerator divided by the denominator is the express quantity of every fraction; therefore, when one fraction is to be divided by another, we take the numerators as the numbers given, which must be cleared from being divided by their denominators, that is, we must take away those divisors, according to the last example in Multiplication; which is done by multiplying each numerator by the others denominator, the products whereof will be the numbers or numerators, cleared for division, and both thereby increased in the same proportion.

H

Suppose

$\frac{1}{2})\frac{3}{4}(\frac{6}{4}=1\frac{1}{2}$ Suppose $\frac{3}{4}$ to be divided by $\frac{1}{2}$, as at the side, here 3 is to be divided by 1; by the fractions $\frac{3}{4}$ and $\frac{1}{2}$, are the quotients of 3 divided by 4, and of 1 divided by 2; therefore, to clear each from being quotients, and to increase them in a like proportion, we multiply 3 by 2, and 1 by 4; which is really multiplying both the given fractions by 8, and produces $\frac{6}{4}$ equal to $1\frac{1}{2}$.

This shews, that when a fraction, or any quantity, is divided by a proper fraction, the quotient must be greater than the dividend, in the same proportion as the divisor is less than an unit.

If a mixed number is given to be divided by a whole number, the integral part of the dividend may first be divided; and, if any thing remains, multiply it by the denominator of the fraction, and add the numerator to the product for a numerator to the fractional part of the quotient, and place the product of the divisor so multiplied for a denominator; but, if such numerator is greater than the denominator, the excess is to be the numerator, and 1 added to the integral part of the quotient.

The DIRECT RULE *of* THREE

IN

VULGAR FRACTIONS.

AFTER the numbers are prepared by reduction, state and work the question by the rules given in whole numbers.

E X A M.

E X A M P L E S.

1st. What will $\frac{1}{2}$ a pound of snuff cost, if 22 lb. $\frac{3}{4}$ of the same come to 7 l. $\frac{1}{3}$?

lb. l. lb.
If 22 $\frac{3}{4}$ cost 7 $\frac{1}{3}$, what will $\frac{1}{2}$?

Prepared by the 2d rule of reduction, it will stand thus:

$$\text{If } \frac{89}{4} \dots \frac{22}{3} \dots \frac{1}{2}$$

then the second and third numbers multiplied together produce $\frac{22}{3}$; which divided by $\frac{89}{4}$, gives $\frac{88}{534}$; which valu'd by the 7th rule of reduction, is 3 s. 3 d. 2 qu.

$\frac{108}{534}$

2^d. Bought 87 gallons and an $\frac{1}{2}$ of brandy for 40 l. at the same rate, what would $\frac{1}{2}$ a gallon cost?

Gall. l. Gall.
If 87 $\frac{1}{2}$ cost 40, what will $\frac{1}{2}$?
prepared $\frac{175}{2} \quad \frac{40}{1} \quad \frac{1}{2}$

$$\text{then } \frac{40}{1} \times \frac{1}{2} = \frac{40}{2} \div \frac{175}{2} = \frac{80}{350} = \frac{8}{35}, \text{ or } 4 \text{ } 6 \text{ } 3 \text{ } \frac{5}{7}$$

Another R U L E.

When your question is stated, and your numbers prepared, multiply the *denominator* of the first number and the *numerators* of the second and third continually, and place the product for a *numerator*: Then multiply the *numerator* of the first number and the *denominators* of the second and third continually, and place the product for its *denominator*; the new fraction so found, is the answer of the question. Take the stating of the last question for an

E X A M P L E.

If $\frac{175}{2} \dots \frac{40}{1} \dots \frac{1}{2}$ facit $\frac{80}{350}$.

For 2, the denominator of the first number, multiply'd by 40 and 1, the numerators of the second and third numbers, is 80.

And 175, the numerator of the first, multiply'd by 1 and 2, the denominators of the second and third, is 350. These products, placed fractionwise, make $\frac{80}{350}$, as before.

3d. What will 82 C. of sugar come to, if $\frac{3}{4}$ of $\frac{3}{8}$ of a C. costs $\frac{9}{10}$ of a pound Sterling?

C.	l.	C.
If $\frac{3}{4}$ of $\frac{3}{8}$ cost $\frac{9}{10}$, what will 82?		
$\frac{9}{10} \dots \frac{82}{1}$		
Facit $\frac{23616}{90}$, or 262 l. 8s.		

More E X A M P L E S.

What will $\frac{1}{4}$ of an ounce of snuff cost, if the lb. comes to 8s. $\frac{1}{3}$?

Answer, 1d. 2qua. $\frac{1}{4}$.

At 1d. $\frac{1}{2}$ per ounce, what will 5 C. $\frac{1}{2}$ come to?

Answer, 6l. 12s.

The INDIRECT RULE of THREE IN VULGAR FRACTIONS.

HERE also you have only to observe the directions given in whole numbers:

Or this R U L E.

After the numbers are stated and prepared, multiply the *numerators* of the first and second numbers and the *denominator* of the third continually for a *numerator*; and the *denominators* of the first and second numbers, and the *numerator* of the third continually for a *denominator*; the fraction so found is the *Answer*. See both ways in the example.

E X A M P L E.

How many yards of stuff, $\frac{1}{2}$ yard wide, are equal to $36 \frac{1}{4}$ yards of $\frac{3}{4}$ wide?

$$\begin{array}{l} \text{If } \frac{3}{4} \text{ ————— } 36 \frac{1}{4} \text{ ————— } \frac{1}{2} \\ \text{Prepared } \frac{3}{4} \dots\dots \frac{145}{4} \dots\dots \frac{1}{2} \\ \frac{3}{4} \times \frac{145}{4} = \frac{435}{16}, \quad \frac{435}{16} \div \frac{1}{2} = \frac{870}{16}, \text{ or } 54 \frac{3}{8}. \end{array}$$

So also 3, 145, and 2, the *numerators* of the first and second numbers, and the *denominator* of the third, multiplied together, is 870.

And 4, 4, and 1, the *denominators* of the first and second, and *numerator* of the third, multiplied together, produce 16.

These numbers, fractionally placed, make $\frac{870}{16}$, as before.

More EXAMPLES.

If *A.* lends *B.* 35 $l.$ $\frac{1}{3}$ for 6 months $\frac{1}{2}$, how long may he keep 10 $l.$ $\frac{1}{4}$ of *B.*'s, to requite himself?

Answer, 22 months, $\frac{50}{123}$.

If *A.* keeps 100 $l.$ $\frac{1}{6}$ of *B.*'s 4 months $\frac{1}{3}$, what sum must *A.* lend *B.* for 2 years $\frac{1}{2}$ to requite him?

Answer, 14 $l.$ $\frac{253}{340}$.

The COMPOUND RULE of THREE
 IN
 VULGAR FRACTIONS.

TO the Rules before given page 42, let it be observed, that where any of the terms given is a fraction (or a mixed number, which must be put down as an improper fraction) in the operation, the denominator will be a factor contrary to the numerator; and when an equation is made for the quotient *Q*, where the several factors of the dividend are put as a numerator, and the several factors of the divisor as a denominator; then whatever factors in each are alike, they may be struck out, and for those equally dividable take their quotients, whereby the operation will frequently be very much shortened.

E X A M P L E.

If 264 men, in $5\frac{3}{4}$ days working $10\frac{1}{2}$ hours each, dig a trench $234\frac{2}{3}$ yards long, $3\frac{3}{4}$ wide, and $2\frac{7}{8}$ deep; in how many days working, $9\frac{3}{4}$ hours each, will 25 men dig a trench $337\frac{1}{2}$ yards long, $4\frac{1}{3}$ wide, and $3\frac{3}{4}$ deep?

men.	days.	hours.	long.	wide.	deep.
If 264	$5\frac{3}{4}$	$10\frac{1}{2}$	$234\frac{2}{3}$	$3\frac{3}{4}$	$2\frac{7}{8}$
25	Q	$9\frac{3}{4}$	$337\frac{1}{2}$	$4\frac{1}{3}$	$3\frac{3}{4}$

Here, as in *Ex. 5.* page 45, the trenches are the produced terms, and the men, days, and hours the producing; therefore

$$Q = \frac{675 \times 13 \times 15 \times 264 \times 23 \times 21 \times 4 \times 3 \times 4 \times 8}{704 \times 15 \times 23 \times 25 \times 39 \times 4 \times 2 \times 2 \times 3 \times 4} = \frac{27 \times 21}{4}$$

$$= \frac{567}{4} = 141\frac{3}{4} \text{ days.}$$

THE
ARITHMETIC
OF
DECIMAL FRACTIONS.

THIS part of Arithmetic is very compendious and easy in operation, and therefore very useful, especially in calculations of interest, valuing annuities, &c. But, by reason some fractions cannot be exactly expressed by decimal parts, it may not always be convenient to work with them. Mr. *Cunn* hath, in his late treatise on fractions, very curiously shewn how they may be used with least loss; but, since such exactness destroys the brevity, I cannot but think, where great perfection is required, vulgar fractions preferable.

C H A P. I.

Of NUMERATION, ADDITION, SUBTRACTION, MULTIPLICATION, *and* DIVISION of DECIMAL FRACTIONS.

N U M E R A T I O N.

IN DECIMAL FRACTIONS, the figures expressed with a point or comma prefixed is the numerator, and 1 with as many cyphers as the numerator hath places is always the denominator, and therefore known without being set down; thus, $.5$ stands for $\frac{5}{10}$, $.25$ for $\frac{25}{100}$, $.123$ for $\frac{123}{1000}$, &c. wherein we imagine the integer or unit (whether it be time, coin, weight, measure, &c. as a year, a pound sterling, a pound weight, a bushel, a yard, a foot, &c.) to be divided into 10, 100, 1000, &c. equal parts at pleasure; for dividing it first into 10 equal parts, and then each of those into 10 lesser equal parts, will divide the unit into 100 equal parts; and if again we divide each of those hundredths into 10 equal parts, the unit will be divided into 1000 equal parts, and so on infinitely.

The different value of the several places will more plainly appear from their explication under the following number.

Integral Parts.						Fraction Parts.						
6	5	4	3	2	1,	2	3	4	5	6	7,	
C	X	T	H	T	U	T	H	T	X	C	M	
Thou-	Thou-	Hund-	Hund-	Tens.	Units.	Tenth-	Hund-	Thou-	Thou-	Thou-	Milli-	
sands.	sands.	reds.	reds.			ths.	reds.	sands.	sands.	sands.	ons.	

From whence it is evident,

That as the significant figures or parts in whole numbers increase by a tenfold proportion towards the left-hand from the units place; so, in Decimal Fractions, they decrease in the same proportion towards the right; therefore cyphers at the left-hand in a decimal fraction decrease the value in a tenfold proportion, by removing those significant figures farther from the unit's place.

Thus $\left\{ \begin{array}{l} .5 \text{ is five tenth parts } = \frac{5}{10} = \frac{1}{2} \\ .05 \text{ is five hundredths } = \frac{5}{100} = \frac{1}{20} \\ .005 \text{ is five thousandths } = \frac{5}{1000} = \frac{1}{200} \end{array} \right\} \text{ of an unit.}$

But cyphers at the right-hand of a decimal fraction alter not its value, because the places of the significant figures thereof are not thereby changed; for .5, .50, .500, &c. are each $\frac{5}{10}$ of an unit, the 5 in each being in the first place of fractions, *viz.* the place of tenths.

These things understood, the rest is easy.

A D D I T I O N.

HAVING set down your proposed numbers, units under units, and fraction places under places of like value, add them as if they were all whole numbers, separating so many fractional places in the total, as were in that given number which had the most.

E X A M.

E X A M P L E S.

Add 5,42, 38,583, 421,06, and ,8345 together.

These numbers, 5,42
rightly placed, 38,583
will stand thus: 421,06
 ,8345

Total, 465,8975

l. *l.* *l.*
 Whats the sum of 4,06, 21,734, and 523,8642
 21,734
 4,06

Answer, 549,65821.

S U B T R A C T I O N.

HAVING placed the numbers as directed in addition, work still as if they were all whole numbers.

E X A M P L E S.

l. *l.*
 From 839,38275 take 10,49561
 10,49561

 828,88714 *Remainder.*

Yards. *Yards.*
 Take 23,279 from 451,15842
 23,279

Remains, 427,87942

MULTIPLICATION.

Here the numbers are to be set down, without regard to the value of their places, and work'd in all respects the same as in whole numbers; only observe to separate so many places of fractional parts at the right hand in the product, as were in both multiplicand and multiplier.

EXAMPLES.

Multiply 2,735641
by 54382

$$\begin{array}{r}
 5471282 \\
 21 \ 885128 \\
 82 \ 06923 \\
 1094 \ 2564 \\
 13678 \ 205 \\
 \hline
 1,4876 \ 9628862
 \end{array}$$

Multiply 73,82564365
by 8,7434592

$$\begin{array}{r}
 14765128730 \\
 66443079285 \\
 3 \ 6912821825 \\
 29 \ 530257460 \\
 221 \ 47693095 \\
 2953 \ 0257460 \\
 51677 \ 950555 \\
 590605 \ 14920 \\
 \hline
 \end{array}$$

Product 645,491503167514080

A compendious way to multiply numbers that have many fractional places; by omitting all the figures to the right of the perpendicular lines in the examples above.

RULE.

Place one of the numbers just as it is given, for a multiplicand, and under that fractional place which you would have be the last in the product, set the unit's place of your other number; then reversing all the other figures, multiply with them in their order, beginning always with that figure of the multiplicand, which stands over the number you work with; having also respect to the increase you think would have been

been brought from the omitted figures on the right hand, had they been multiplied.

Note, The usual allowance for such omitted figures is as follows, *viz.* If the next figure on the right hand of that you begin with in the multiplicand, multiply'd into the figure of the multiplier you are working with, gives a product between 5 and 15, carry 1; if the product be above 15, and less than 25, carry 2; and if it arise to any number between 25 and 35, carry 3, &c.

To prove the certainty of the rule, we will take the same examples before given.

EXAMPLES.

Multiply 2,735641 by ,54382, and produce only four places of decimals.

$$\begin{array}{r} 2,735641 \\ 28345,0 \\ \hline \end{array}$$

13678 *the Increase here carried is 3 for 5x6*

1094 *the Increase here is 2 for 4x5*

82 *the Increase here is 1 for 3x3*

22 *the Increase here is 6 for 8x7*

$$\begin{array}{r} 1,4876 \\ \hline \end{array}$$

Note, There being no units in the multiplier, its place is supply'd with a cypher.

Multiply 73,82564365 by 8,7434592, and produce but 3 places of decimal fractions.

$$\begin{array}{r} 73,82564365 \\ 2954347,8 \\ \hline \end{array}$$

$$\begin{array}{r} 590605 \\ \hline \end{array}$$

$$\begin{array}{r} 51678 \\ \hline \end{array}$$

$$\begin{array}{r} 2953 \\ \hline \end{array}$$

$$\begin{array}{r} 222 \\ \hline \end{array}$$

$$\begin{array}{r} 29 \\ \hline \end{array}$$

4 *the Increase carried for 5x7*

$$\begin{array}{r} 645,491 \\ \hline \end{array}$$

Note,

Note, To multiply any decimal fraction by 10, 100, 1000, &c. is only to remove the mark of separation so many places towards the right hand as there are cyphers.

$$\text{Thus, } 8,3564537 \left\{ \begin{array}{l} 10, \\ 100, \\ 1000, \\ 10000, \end{array} \right\} \text{ is } \left\{ \begin{array}{l} 83,564537 \\ 835,64537 \\ 8356,4537 \\ 83564,537, \&c. \end{array} \right.$$

D I V I S I O N.

THIS rule also being work'd the same as in whole numbers, the only difficulty is to value the quotient; which may be done by either of the following general rules, *viz.*

R U L E 1.

The first figure in the quotient is always to stand in the same place with that figure of the dividend, which answers, or stands over the place of units in the divisor. Or,

R U L E 2.

The quotient must always have so many fractional places, as the dividend has more than the divisor.

Note, 1st. If the divisor and dividend have both the same number of fraction places (as in the second example) the quotient will be all a whole number.

2d. If the dividend hath not so many places of fractions as are in the divisor, (as in example the 3d.) so many cyphers must be annex'd to the dividend, as will make them equal; and the quotient will then, as before, all be a whole number.

3d. But if, when division is done, the quotient has not so many figures as it should have fraction places (as in the last example) so many cyphers must be prefix'd, as there are places wanting.

E X A M-

EXAMPLES.

1st. Divide 10,55439 by 4,569

$$\begin{array}{r}
 4,569 \overline{) 10,55439} (2,31 \\
 \underline{14163} \\
 4569 \parallel \\
 \dots
 \end{array}$$

Here, according to the first rule, 4, the unit's place of the divisor, standing under units, or the first place of whole numbers in the dividend; 2, or first figure plac'd in the quotient, must also be made the first place of whole numbers, by placing the point of separation immediately after it.

Or, according to the second rule, there being five places of fractions in the dividend, and but three in the divisor; two places must be separated off in the quotient, to make up the number.

Ex. 2^d. Divide 85643,825 by 6,321.

$$\begin{array}{r}
 6,321 \overline{) 85643,825} (13549 \\
 \underline{22433} \\
 34708 \\
 \underline{31032} \\
 57485 \\
 \underline{596}
 \end{array}$$

Here 6, the place of units in the divisor, standing under 8, the fifth place of the whole numbers in the dividend,

dividend, 1, the first figure of the quotient, must also stand in the same place, which makes all the quotient whole numbers.

And so it must also be by the second rule, because the dividend has no more places of fractions than the divisor.

Ex. 3. Divide 7382,54 by 6,4352.

$$6,4352)7282,5400(1147$$

Here there not being so many places of fractions in the dividend as in the divisor, two cyphers are added to make them even; and being so, the quotient is all a whole number as before.

Ex. 4. Divide ,08516438 by 423.

$$423)0,08516438(0,00020133$$

$$\begin{array}{r} \cdot 564 \\ \hline \end{array}$$

$$\begin{array}{r} 1413 \\ \hline \end{array}$$

$$\begin{array}{r} \cdot 448 \\ \hline \end{array}$$

$$179$$

In this example, the quotient consisting but of five figures, whereas there should, by the rules, be eight fraction places of decimal parts, three cyphers are prefixed to make up the number, as was directed in *Note the third.*

There

There is also a way of contracting the work of a large division, somewhat like that compendious method before taught, of multiplying numbers of many places of decimal fractions, and bringing out only so many in the product as are necessary for the answer required.

R U L E.

Having consider'd, by the first rule given for valuing the quotient, in what place the first figure thereof ought to stand, and accordingly determined how many figures it is necessary it should consist of, you may from thence judge whether any, or how many of the right hand figures of your divisor may be neglected; so that reserving only so many figures of the dividend as are necessary for once answering the divisor, leave out the rest to the right hand as useless; and having set the proper figure in the first place of the quotient, work with it as usual; then omitting the right hand figure of your divisor, seek how often the other figures of your divisor are contained in the remainder; which figure being entered in the quotient, and work'd with as usual in division, (with regard, however, to the carriage that would have been brought from the omitted figures, as before directed in multiplication) thus continuing to neglect the right hand figure (putting a point under for ready knowing it) of your divisor every time you seek a new quotient figure, you will still be able to divide the remainder left after the last subtraction by the divisor so lessened, till you have brought out the determin'd number of figures in the quotient.

The following example, which is set down both at length, and the contracted way, will make all clear.

EXAMPLE.

3.4637528	95,4327	56463275	(27,55188
	692750	56	
			3.46375)95,4327 27,5518
261577	004		69 2750
242462	696		
			26 1577
19114	3086		24 2463
17318	7640		
			19114
1795	54463		17319
1731	87640		
			1795
63	668232		1732
34	637528		
			63
29	0307047		35
27	7100224		
			28
1	32068235		27
			1

And thus are all the figures on the right hand of the perpendicular line in the example work'd at length, sav'd by working after this contracted manner.

As multiplying by 10, 100, 1000, &c. is only removing the separating point of the multiplicand, so many places towards the right hand as there are cyphers in the multiplier; so, to divide by 10, 100, 1000, &c. is only removing the mark of separation, after the same manner, towards the left hand.

Thus 83564,537 $\left\{ \begin{array}{l} 10, \\ 100, \\ 1000, \\ 10000, \end{array} \right\}$ is $\left\{ \begin{array}{l} 8356,4537 \\ 835,64537 \\ 83,564537 \\ 8,3564537 \end{array} \right.$
divided by

These examples being, I hope, sufficient to make division plain, we will now proceed to *Reduction of Decimals*.

CHAP. II.

Of REDUCTION of DECIMALS.

THERE are two sorts of *Reduction of Decimal Fractions*. 1st. Changing a vulgar fraction to a decimal. 2. Valuing a decimal fraction by the known parts of an integer.

1st. To reduce or change a vulgar fraction to a decimal.

R U L E.

Place cyphers at pleasure after the *numerator*, and divide by the *denominator*.

E X A M P L E S.

Reduce $\frac{1}{4}$ to a decimal, 4)1,00(,25 *facit*.

What decimal parts are equal to $\frac{3}{8}$?

8)3,000(,375 *facit*.

And so any part of coin, weight, measure, or time, &c. may be brought into a decimal, by first reducing it into a vulgar fraction (which is done by placing the known parts of the integer as a denominator under the parts given) then annexing cyphers to the *numerator*, and dividing by the *denominator*, as before directed.

E X A M.

E X A M P L E S.

What decimal parts of a pound are equal to 16 s. ?

i. e. $\frac{16}{20}$ 20)16,0(,8 *facit*

What decimal parts of a lb. *Troy*, are equal to 9 oz. ?

i. e. $\frac{9}{12}$ 12)9,00(,75 *facit*.

Note, If the given parts are of several denominations, they may be reduced either by so many distinct operations as there are different parts, or by first reducing them into their lowest denomination, and then one division will serve. See both ways in the

E X A M P L E S.

Reduce 14 s. 9 d. $\frac{3}{4}$ into decimal parts of a pound Sterling: By the first way there will be these three vulgar fractions to be chang'd, $\frac{14}{20}$, $\frac{9}{240}$, and $\frac{3}{960}$.

20)14,00(,70	240).90000(,0375	,70
	1800	,0375
960)3,0000(0031	1200	,0031
1200	1200	<i>facit</i> ,7406
240	.00	

s. d.

The other way. 14 9 $\frac{3}{4}$ being reduced to farthings.

12

177

4

will be 711

which divided
by its deno-
minator,

960)711,0000(,7406 is produc'd as
(before.

3900

.6000

240

What

What decimal parts of C. wt. are equal to 3 qrs. 18 lb. ?

By the first way :

$$\begin{array}{r} \frac{3}{4} \text{ and } \frac{18}{112} \quad 4)3,00,75 \\ \quad \quad \quad ,1607 \\ \hline \text{Answer, ,9107} \end{array} \quad \begin{array}{r} 112)18,0000,1607 \\ \hline \cdot 680 \\ \hline \cdot \cdot 800 \\ \hline 16 \end{array}$$

By the second way.

$$\begin{array}{r} \text{qrs. lb.} \\ 3 \quad 18 \\ 28 \\ \hline 102 \\ \hline 112 \text{ Denominator.} \end{array} \quad \begin{array}{r} \text{Answer, } 112)102,0000,9107 \\ \hline \cdot 120 \\ \hline \cdot \cdot 800 \\ \hline 16 \end{array}$$

There is also another way of reducing parts of different denominations into decimals of the highest integer ; thus,

R U L E.

Bring the lowest parts into decimals of the next superior denomination ; and on the left hand of the decimal fraction found, place the parts given of that said next superior denomination ; so proceeding, till you bring out the decimal parts of the highest integer requir'd, by still dividing the product by the next superior denominator For examples, take those before given, viz.

Reduce

Reduce 14 s. 9 d. $\frac{3}{4}$ into decimal parts of a pound Sterling.

$$\begin{array}{r|l}
 4 & 3, \\
 \hline
 12 & 9,75 \\
 \hline
 20 & 14,8125 \\
 \hline
 & ,7406
 \end{array}$$

Here the 3 farthings being divided by 4, gives 75, to which the 9 d. being prefixed, it becomes 9,75; which divided again by 12, quotes ,8125; before which the 14 s. being also placed, it makes 14,8125; which being lastly divided by 20, the quotient ,7406, is the answer.

What decimal parts of a C. wt. are equal to 3 qrs. 18 lb.?

$$\begin{array}{r|l}
 28 \left\{ \begin{array}{l} 7 \\ 4 \\ 4 \end{array} \right. & 18, \\
 \hline
 & 2,5714 \\
 \hline
 & 3,6428 \\
 \hline
 & ,9107 \text{ Answer.}
 \end{array}$$

Note, Instead of dividing by 28, which would be troublesome; 'tis better to divide by 7 and 4, as in the example, 4 times 7 being 28.

Thus you see how easily the decimals, answering to any given parts of coin, weight, measure, &c. are found; and after the same manner are the following tables formed.

But shillings, pence, and farthings, may more readily be reduc'd thus:

R U L E.

For the shillings (if their number be even) set down their half in the first place of decimal fractions, and let the second and third places be fill'd up with the farthings contain'd in the remaining pence and farthings, always remembring to add one, when the number

ber is or exceeds 25 ; but if the number of shillings be odd, the second place must also be increased by 5.

Thus 8 s. 5 d. $\frac{1}{2}$, reduced to a decimal fraction, is ,422 ; 4 in the first place standing for 8 s. and 22 farthings being 5 d. $\frac{1}{2}$.

Again, 8 s. 6 d. $\frac{1}{2}$ will be ,427.

1 being added to 26, the number of farthings in 6 d. $\frac{1}{2}$, because exceeding 25.

And lastly, 9 s. 6 d. $\frac{1}{2}$, express'd decimally, is ,477.

5 being added to the second place, because the number of shillings is odd, 1 shilling being ,05.

The second sort of reduction is, to value a decimal fraction by the known parts of its integer ; which is thus done :

R U L E.

Multiply the decimal fraction given by the known parts of the next inferior denomination, and separating the product as directed in multiplication, then multiply the decimal fraction of the product by the known parts of the next inferior denomination ; thus proceeding till you have brought it into the least known parts of the integer, the figures standing on the left of the separating points will be the parts required.

E X A M P L E S.

Ex. 1. What's the value of ,7691, the integer a pound sterling? (i. e.) What shillings, pence, and farthings are equal to ,7691 parts of a pound sterling?

	,7691 parts of a pound
<i>multiplied by</i>	20 the shillings in 1 l.

	15,3820 and parts of a skill.
<i>produce shillings</i>	
<i>which multiplied by</i>	12 the pence in 1 skill.

	4,5840 and parts of a penny.
<i>make pence</i>	
<i>which again multiplied by</i>	4 the farthings in 1 d.

<i>give farthings</i>	2,3364 and part of a farth.
-----------------------	-----------------------------

So

So that the value of ,7691 decimal parts of a pound sterling is 15 s. 4 d. $\frac{1}{2}$.

Ex. 2. What ounces, penny weights, and grains, are equal to ,84362 decimal parts of a pound Troy?

	,84362 parts of a pound Troy,
multiplied by	12 the ounces in one pound,
give ounces	10,12344 and parts of 1 ounce.
multiplied by	20 the penny-weights in 1 oz.
give penny-weights	2,46880 and parts of 1 penny-wt.
multiplied by	24 the grains in 1 penny-wt.
	187520
	93760

Give grains 11,25120 parts of 1 grain.

The Answer therefore is 10 oz. 2 dwts. 11 grs.

But the decimal parts of a pound sterling may be thus valued at sight :

R U L E.

The figure standing in the first place of decimal parts, doubled, gives shillings ; but if the figure in the second place is or exceeds 5, one more must be added to their number ; the second figure, (if under 5) or its excess, (if above 5) join'd with the third, are so many farthings ; only remember to abate 1, if their number amounts to near 25 ; or 2, if near 50.

The reason of this abatement is, that as 1000 is the denominator of every decimal fraction consisting of 3 places, so consequently by reckoning the figure standing in the said third place, as so many farthings, we thereby allow 1000 farthings to the pound ; whereas indeed there are

are but 960: The overplus therefore being 40 in 1000, is 4 in a 100, or 1 in 25, &c. according to the direction above.

EXAMPLES.

Ex. 1. What's the value of ,375 parts of a pound Sterling?

Answer, 7 s. 6 d.

The 3 doubled is 6 shillings; to which 1 s. being added for the 5, in the second place, makes it 7 s. and 2 remaining, join'd with the 5 in the 3d place, being accounted 25 farthings, from which 1 being deducted, there remain 24 farthings, or 6 pence.

Ex. 2. What's the value of ,846 parts of a pound?

Answer, 16 s. 11 d.

The 8 doubled, making 16 s. and 2 abated from the 46 farthings, leaves 44 farthings, or 11 d.

Thus are shewn the ways both of finding the decimal fractions answerable to any given parts of coin, weight, measure, &c. or the value of any given decimal in the known parts of its integer.

I have likewise annexed TABLES ready calculated by the foregoing rules, answering to the same ends.

The use of which is very plain ; one column in each table containing the number of shillings, ounces, drachms, pints, bushels, days, hours, or any thing else you may want ; and against the said number, in the other column, the decimal parts answering : Thus against 3 pence in the first table, is found ,0125, and against 40 days in the fourth table, 109589.

But if the number you want is not to be found in the said tables (many compound numbers having been omitted, to reduce them into so narrow a compass) you must add the parts of such two or three numbers together, as will compose it, and the total will be the decimal fraction sought. Thus, if the decimal fraction answering to 17 pennyweights, the integer a pound *Troy*, were to be sought in the 2d table, ,04166, the decimal fraction answering to 10 penny-weights, and ,029166, that answering to 7 penny-weights, being added together, the total ,070832, will be found the answer ; or, if in table the fourth, you would find the decimal of 352 days,

$$\begin{array}{r}
 \text{add } ,821918 \\
 ,136986 \\
 ,005479 \\
 \hline
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{the decimal} \\ \text{fractions of} \end{array} \left\{ \begin{array}{l} 300 \\ 50 \\ 2 \end{array} \right\} \text{Days.}$$

$$\text{total } ,964383 \text{ answering } 352 \text{ Days.}$$

And so of any other.

Decimal Tables

Coin & Weight

Table 1 st				Table 2 ^d		Table 3 ^d	
Coin 1 st ster: 4 th Integer				Troy Weight 1 st lb. the Integer		Avoirdupoise 12 lb. 4 th Integer.	
S	Dec:	S	Dec:	Ounces	Decim:	Quarters	Decim:
19	.95	9	.45	11	.916666	3	.75
18	.9	8	.4	10	.833333	2	.5
17	.85	7	.35	9	.75	1	.25
16	.8	6	.3	8	.666666	Pounds	Decim:
15	.75	5	.25	7	.583333	20	
14	.7	4	.2	6	.5	10	
13	.65	3	.15	5	.416666	9	
12	.6	2	.1	4	.333333	8	
11	.55	1	.05	3	.25	7	
10	.5			2	.166666	6	
				1	.083333	5	
				Note This Table of Oz: will also serve for Inches Months or Days		4	.04643
				Penny n ^o		3	.035714
						2	.026786
						1	.017857
							.008928
Pence	Decim:			10	.041666	Ounces.	Decim:
11	.045833			9	.0375	10	.003580
10	.041666			8	.033333	9	.003022
9	.0375			7	.029166	8	.004464
8	.033333			6	.025	7	.003906
7	.029166			5	.020833	6	.003348
6	.025			4	.016666	5	.002790
5	.020833			3	.0125	4	.002232
4	.016666			2	.008333	3	.001673
3	.0125			1	.004166	2	.001116
2	.008333					1	.000558
1	.004166						
				Grains	Decim:	Drams	Decim:
				20	.003472	10	.000348
				10	.001736	9	.000313
				9	.001562	8	.000279
				8	.001389	7	.000244
				7	.001216	6	.000209
				6	.001042	5	.000174
				5	.000868	4	.000139
				4	.000694	3	.000104
				3	.000521	2	.000069
				2	.000347	1	.000034
				1	.000173	$\frac{1}{2}$	.000017
				$\frac{1}{2}$	.000086		
Farthings	Decim:						
3	.003125						
2	.002083						
1	.001042						

Webster script.

Bickham Sculpit.

Decimal Tables

of Measure and Time

Table 4. th		Table 5. th			Table 6. th	
Time 1 year of Integer		Measure Liquid. Dry. Integer			Time 1 Day of Integer	
Days	Decim:	1 Gall:	1 Quarter		Hours	Decim:
300	.821918	Pints	Decim:	Bush	20	.833333
200	.547945	7	.875	7	10	.416666
100	.273973	6	.75	6	9	.375
90	.246375	5	.625	5	8	.333333
80	.219178	4	.5	4	7	.291666
70	.191781	3	.375	3	6	.25
60	.164383	2	.25	2	5	.208333
50	.136986	1	.125	1	4	.166666
40	.109589	q ^{rs}	Decim:	Pecks	3	.125
30	.082192	3	.09375	3	2	.083333
20	.054794	2	.0625	2	1	.041666
10	.027397	1	.03125	1		
9	.024657	Decim:	q ^{rs} Pecks		Minutes	Decim:
8	.021918	.023437	3		50	.034722
7	.019178	.015625	2		40	.027777
6	.016438	.007812	1		30	.020833
5	.013699	Decim:	Pints		20	.013888
4	.010959	.005859	3		10	.006944
3	.008219	.003906	2		9	.00625
2	.005479	.001953	1		8	.005555
1	.002739				7	.004861
		Cloth Measure			6	.004166
		1 Yard of Integer			5	.003472
		Quarters	Decim:		4	.002777
		3	.75		3	.002083
		2	.5		2	.001388
		1	.25		1	.000694
		Nails	Decim:			
		3	.1875			
		2	.125			
		1	.0625			
Wetster scrip ^t					Bickham foul ^t	

C H A P. III.

Of the RULE of THREE.

THIS *Rule of PROPORTION* having been already sufficiently explained, both as to its nature and operation, in *Vulgar Arithmetic*, 'tis needless to repeat here what has been there said. Only therefore observe, that instead of preparing your numbers, by reducing them into their lowest denomination, you must now bring their fractional parts into decimals. See the examples.

E X A M P L E S.

Ex. 1. If the price of 26 yards and $\frac{1}{2}$ of drugget is 3 *l.* 16 *s.* 3 *d.* what will 32 yards $\frac{1}{4}$ come to?

The fractional parts of the numbers being reduced to decimals, and the question stated as usual, the work will stand as follows :

Yards. l. Yards.
If 26,5—3,8125—32,25

32,25
—
190625
76250
76250
—
114375

26,5)122,953125(4,63974
1695

—
1053

—
2581

—
1962

—
1075

—
15

*The quotient valued
by the short rule for
money, makes 4l. 12s.
9d. $\frac{1}{2}$.*

*Ex. 2. What will the pay of 540 men amount to,
at 1 l. 5 s. 6 d. per man?*

Man. l. Men.
If 1—1,275—540

540

—
51000
6375

Answer, 688,500 or 688 l. 10s.

*For more examples, take those given in Vulgar
Arithmetic.*

The

The Indirect RULE of THREE.

THE Indirect RULE of THREE is also the same here as in *Vulgar Arithmetic*, differing only in the preparation of the numbers.

EXAMPLE.

What must the penny-loaf weigh, when wheat is 10s. the bushel, if, when at 6s. 5d. per bushel, the penny-loaf weighs 5 oz. $\frac{1}{2}$?

$$\begin{array}{r} \text{If, } 32083 \text{ ————— } 5,5 \text{ ————— } ,5 \\ \quad \quad \quad 5,5 \\ \quad \quad \quad \hline \quad \quad 160415 \\ \quad \quad 160415 \\ \quad \quad \hline ,5) 1,764565 \text{ (oz. dwts. grs.} \end{array}$$

Answer, $3,52913 = 3 \dots 10 \dots 14$ nearly.

Note, Where the last figure of a Decimal Fraction would recur infinitely, and it be required to have the operation exact, annex the Vulgar Fraction in that place, *viz.* that figure numerator with 9 denominator or their equal, and in any multiplication thereof, add such part of the multiplier in the lowest place to make up the product. Suppose in the above example,

$$\begin{array}{r} \text{If, } 3208 \frac{2}{9} = 3208 \frac{2}{9} \text{ ————— } 5,5 \text{ ————— } ,5 \\ \quad \quad \quad 5,5 \\ \quad \quad \quad \hline \quad \quad 16040 \\ \quad \quad 16040 \\ \quad \quad \hline \quad \quad 18 \frac{1}{9} \\ ,5) 1,76458 \frac{1}{9} \text{ oz. dwts. grs.} \end{array}$$

Answer, $.3,5291 \frac{2}{9} \text{ oz.} = 3 \dots 10 \dots 14$ more near.

Mr. Gardiner has shewn, in the explication of his Tables of Logarithms, a very easy method of getting the logarithm of any number whose last expressed figure, or figures, would recur infinitely perfect in the lowest place of the tables, upon the foundation of that figure or figures being numerator with 9, 99, 999, &c. denominators, that is, with as many nines as there are recurring figures.

The Double or Compound RULE of THREE.

IF the carriage of $\frac{1}{2}$ a C. wt. 40 miles comes to 6d. what will be the charge of carrying 16 C. $\frac{1}{4}$ 100 miles?

First, If, 5—,025—16,25
 ,025

8125
3250

5) 40625(

$$\begin{array}{r} \text{Miles.} \quad \underline{\hspace{2cm}} \quad \text{Miles.} \\ \text{Then, If } 40 \text{ --- } 8125 \text{ --- } 100 \\ \hspace{10cm} 100 \end{array}$$

40) 81,2500(

Answer, 2,03125 l. = 2 l. 0 s. 7 d. $\frac{1}{2}$

Or thus,

$$\begin{array}{r} \text{C.} \qquad \qquad \text{I.} \qquad \qquad \text{C.} \\ \text{If } 5 \text{ --- } 025 \text{ --- } 16,25 \\ \underline{40 \text{ Miles.}} \qquad \qquad \underline{100 \text{ Miles.}} \\ 20,0 \qquad \qquad 1625,00 \end{array}$$

$$\begin{array}{r} 1625,00 \\ ,025 \\ \hline 8125 \end{array}$$

3250

20,)40,625

Answer, $\overline{203125} l. = 2 l. 0 s. 7 d. \frac{1}{2}$

Or thus,

<i>C.</i>	<i>Miles.</i>	<i>l.</i>
If, 5 carried	40 comes to	,025
16,25	100	Q.

$$Q = \frac{0,025 \times 100 \times 16,25}{0,5 \times 40} = \frac{0,05 \times 10 \times 16,25}{4} = 2,03125$$

CHAP. IV.

Of PRACTICE.

THE fractions of some questions in this rule may with advantage be worked decimally; as,

1st. When the given price is just two shillings, 'tis only separating the right-hand figure for a decimal fraction, the rest are pounds:

EXAMPLE.

5295 lb. at 2 s. per lb. comes to $\left\{ \begin{array}{l} 529,5 \\ \text{or} \\ 529 \text{ l. } 10 \text{ s.} \end{array} \right.$

2^d. When the price is any *aliquot* part of 2 s. it is only separating the right-hand figure, as before, and then dividing the number by such part.

EXAMPLE.

oz. d.
258,3 at 8 per oz.

8 d. $\frac{1}{3}$ 86,1 = 86 l. 2 s. Answer.

3^d. When the given price can be divided into *aliquot* parts of two shillings, separate as before, and take such parts.

E X A M P L E.

	l.	d.
	563,8 at 9 per lb.	
d.		
	6 $\frac{1}{4}$	140,95
	3 $\frac{1}{2}$	70,475

Answer, 211,425 = 211 l. 8 s. 6 d.

4th. When the price is any number of shillings whatsoever, 'tis only multiplying by their half (which is the decimal fraction for them) and the product is the answer.

E X A M P L E S.

lb.	s.	oz.	s.
7296 at 12 per lb.		8264 at 15 per oz.	
,6		,75	
<i>Ans.</i> 4377,6 = 4377 l. 12 s.		41320	
		57848	

Answer, l. 6198,00

5th. The required price of any quantity may be found, by multiplying the said quantity by the decimal fraction answering to the price given.

E X A M P L E.

lb.	l.	s.	d.
8397 at 5 13 6 per lb.			
5,675			
41985			
58779			
50382			
41985			

Answer, 47652,975 = 47652 l. 19 s. 6 d.

6th. If

6th. If there are fractional parts in the given quantity, they must be reduced to decimals, as well as the fractional parts of the price.

E X A M P L E S.

lb.	s.	C.	qrs.	lb.	l.	s.	d.
368 $\frac{1}{2}$	at 13	per lb.	7368	3	14	at 1	12 6
	368,5					7368,875	
	,65					1,625	
	18425					36844375	
	22110					14737750	
						44213250	
Answer,	239,525					7368875	
Equal to	239 l. 10 s. 6 d.						

Answer, 11974,421875
Equal to 11974 l. 8 s. 5 d. $\frac{1}{4}$

The questions proposed in *Vulgar Arithmetic*, may serve for more examples.

C H A P. V.

Of INTEREST and REBATE.

THESE rules having been already sufficiently explain'd in *Vulgar Arithmetic*, it only now remains to shew how much more advantageously they may be work'd by decimal fractions. And, first,

SIMPLE INTEREST.

1st. THE annual interest of any sum of money is found by only multiplying the given principal by the interest of one pound for a year ; which,

at $\left\{ \begin{array}{c} 4 \\ 5 \\ 6 \end{array} \right\}$ per Cent. is $\left\{ \begin{array}{c} ,04 \\ ,05 \\ ,06 \end{array} \right\}$

found by dividing the rate of interest by 100.

E X A M P L E S.

1st. What's a year's interest of 536 at 5 per Cent?

$$\begin{array}{r} .05 \\ \hline \text{Answer, } 26.80 = 26\text{ l. } 16\text{ s.} \end{array}$$

2d. What will the interest of 2367 *l.* come to in a year, at 6 *l. per Cent. per Ann*? ,06

Answer, $\overline{142,02l.} = 142l. \text{ or } 5d.$

2d. Thus a year's interest of any sum being found, 'tis only multiplying that sum by 2, 3, 4, &c. and the product is the interest of the said sum, for 2, 3, 4, or more years.

E X A M P L E.

1.
What's the interest of 623 for 3 years, at 4 per Cent?

$$\begin{array}{r} .04 \\ \hline 24,92 \end{array} \text{ Interest for 1 year.}$$

Answer, $\overline{74,76} = 74\text{l. } 15\text{s. } 2\text{d. } \frac{1}{2}$ for 3 years.

3d. Again, If the time required be months, 'tis only dividing the year's amount by such parts as the given months are of a year.

E X A M -

EXAMPLES.

Ex. 1st. What is the interest of 539 *l.* for four months, at 5 *l.* per Cent. per Annum?

$$\begin{array}{r} l. \\ 539 \\ ,05 \\ \hline 26,95 \end{array}$$
Interest for a year.

Months. $4 \frac{1}{3}$ $8,9833 = 8 \text{ l. } 19 \text{ s. } 8 \text{ d. Answer.}$

Ex. 2d. What will the interest of 729*l.* 10*s.* 6*d.* come to in a year and 5 months, at 6 per Cent. per Annum?

Note, If there are any odd shillings, pence, and farthings, in the principal sum, they must be reduced to decimals; as in this example.

$729,525$
 $,06$

Months 43,77150 *Interest for a year.*
 $4 \frac{1}{3}$ 14,59050 *Interest for 4 months.*
 $1 \frac{1}{4}$ 3,647625 *Interest for 1 month.*

Answer, 62,009625 *Total Interest.*
or
 62 l. 0s. 2 d. $\frac{1}{4}$.

4th. But if it be required to find the interest of a sum of money for any number of days, you must either multiply the year's amount by the number of days proposed, and divide the product by 365, or (which will be sometimes shorter) divide the year's amount by 365, and

and multiply the quotient by the number of days proposed.

E X A M P L E.

What interest will 563 *l.* amount to in 126 days, at 6 *l. per Cent. per Annum*?

$ \begin{array}{r} \text{£} \\ 563 \\ ,06 \\ \hline 33,78 \\ 126 \\ \hline 20268 \\ 6756 \\ 3378 \\ \hline 365)4256,28(11,661 \\ \hline \cdot 606 \\ \hline 2412 \\ \hline 2228 \\ \hline \cdot 380 \\ \hline \cdot 15 \end{array} $	$ \begin{array}{r} 365)33,78000(,09254 \\ \hline \cdot \cdot 930 \\ \hline 2000 \\ \hline 1750 \\ \hline 290 \\ ,09254 \\ 126 \\ \hline 55524 \\ 18508 \\ 9254 \\ \hline 11,66004 \end{array} $
---	---

But for the more ready casting up the interest of money for days, it will be necessary to have the interest of 1 *l.* for 1 day, at all rates ready calculated, as a table or standard. Thus:

The INTEREST of 1 l. for 1 Day.

At {	1	per Cent. is {	,00002739726
	2		,00005479452
	3		,00008219178
	4		,00010958904
	5		,00013698630
	6		,00016438356
	7		,00019178082
	8		,00021917808
	9		,00024657534
	10		,00027397260

The interest of 1 l. for one day, is thus found; the given rate or interest of 100 l. for a year, being divided by 100, quotes the interest of 1 l. for a year; which again divided by 365, gives the required interest of 1 l. for 1 day; which, when multiply'd into both the number of days, and the principal sum, produces the interest for the time required.

E X A M P L E.

What's the interest of 560 l. for 60 days, at 5 l. per Cent. per Annum?

,0001369863
560 Principal.

82191780
6849315

,0767123280
60 Days.

Answer 4,6027396800 = 4 l. 12 s. 0 d. $\frac{1}{2}$.

The

The year being divided into 12 unequal months, the following verses may serve as a *memorandum* of the number of days in each, *viz.*

*Thirty days hath September,
April, June, and November;
February hath twenty-eight alone,
And all the rest have thirty-one.*

But without any farther trouble, the opposite table shews, by inspection only, the number of days from any day in any month, to the same day in any other month. As suppose the number of days, betwixt the 4th of *February* and the 4th of *September*, were requir'd; looking in the column under *February* for *September*, against that month will stand 212, the number of days betwixt the said times: So again, from the 8th of *June* to the 8th of *October*, appears by the table to be 122 days, that being the number against *October*, in the column under *June*.

Note, If the given days are different, 'tis only adding, or subtracting their inequality to, or from the tabular number. Thus had the first example been from the 4th of *February* to the 8th of *September*, it had been 4 days more than 212, *viz.* 216; or had the time, in the last example, been from the 8th of *June* to the 4th of *October*, it had been 4 days less than 122, *viz.* 118.

Note also, If the time exceeds a year, 365 days must be added. Thus from the 4th of *February* 1734, to the 4th of *September* 1735, will be found to be 577 days, the sum of 212, and 365. See the Table.

A Table

Shewing the Number of Days from any day in any Month to the same day in any other Month.

From	Jan:	Feb:	Mar:	April:	May:	June:
	Feb: 31	Mar: 28	April: 31	May: 30	June: 31	July: 30
	Mar: 39	April: 59	May: 61	June: 61	July: 61	Aug: 61
	April: 90	May: 89	June: 92	July: 91	Aug: 92	Sep: 92
	May: 120	June: 120	July: 122	Aug: 122	Sep: 123	Oct: 122
	June: 151	July: 150	Aug: 153	Sep: 153	Oct: 153	Nov: 153
	July: 181	Aug: 181	Sep: 184	Oct: 183	Nov: 184	Dec: 183
	Aug: 212	Sep: 212	Oct: 214	Nov: 214	Dec: 214	Jan: 214
	Sep: 243	Oct: 242	Nov: 245	Dec: 244	Jan: 245	Feb: 245
	Oct: 273	Nov: 273	Dec: 275	Jan: 275	Feb: 276	Mar: 273
	Nov: 304	Dec: 303	Jan: 306	Feb: 306	Mar: 304	April: 304
	Dec: 334	Jan: 334	Feb: 337	Mar: 334	April: 335	May: 334
	Jan: 365	Feb: 365	Mar: 365	April: 365	May: 365	June: 365

From	July:	Aug:	Sep:	Oct:	Nov:	Dec:
	Aug: 31	Sep: 31	Oct: 30	Nov: 31	Dec: 30	Jan: 31
	Sep: 62	Oct: 61	Nov: 61	Dec: 61	Jan: 61	Feb: 62
	Oct: 92	Nov: 92	Dec: 91	Jan: 92	Feb: 92	Mar: 90
	Nov: 123	Dec: 122	Jan: 122	Feb: 123	Mar: 120	April: 121
	Dec: 153	Jan: 153	Feb: 153	Mar: 151	April: 151	May: 151
	Jan: 184	Feb: 184	Mar: 181	April: 182	May: 181	June: 182
	Feb: 215	Mar: 212	April: 212	May: 212	June: 212	July: 212
	Mar: 243	April: 243	May: 242	June: 243	July: 242	Aug: 243
	April: 274	May: 273	June: 273	July: 273	Aug: 273	Sep: 274
	May: 304	June: 304	July: 303	Aug: 304	Sep: 304	Oct: 304
	June: 335	July: 334	Aug: 334	Sep: 335	Oct: 334	Nov: 335
	July: 365	Aug: 365	Sep: 365	Oct: 365	Nov: 365	Dec: 365

1850

Year	Month	Day	Hour	Minute	Second	Latitude	Longitude	Altitude	Barometer	Thermometer	Wind	Clouds	Remarks
1850	Jan	1	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	2	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	3	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	4	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	5	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	6	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	7	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	8	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	9	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	10	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	11	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	12	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	13	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	14	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	15	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	16	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	17	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	18	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	19	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	20	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	21	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	22	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	23	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	24	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	25	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	26	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	27	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	28	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	29	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	30	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear
1850	Jan	31	10	15	30	45° 30'	122° 30'	1000	30.0	60	N	0	Clear

COMPOUND INTEREST.

1st. TO find the amount of any sum, at any rate of compound Interest, for any number of years.

R U L E.

Multiply the amount of 1*l.* for a year, (which at 6 *per Cent.* is 1,06, at 5 *per Cent.* 1,05, &c.) so often into itself, as are the number of years proposed, wanting one; and the last product multiply'd by the principal, will give the amount required.

E X A M P L E.

What's the amount of 500*l.* forborn four years, at 4*l. per Cent. per Annum?*

$$\begin{array}{r}
 1,04 \\
 1,04 \\
 \hline
 1,0816 \\
 1,04 \\
 \hline
 1,124864 \\
 1,04 \\
 \hline
 1,16985856 \\
 500 \\
 \hline
 \end{array}$$

Answer. 584,92928000*l.* = 584*l.* 18*s.* 7*d.*

And thus is the first interest-table (which shews the amount of 1*l.* for any number of years, under 33, at the rates of 5 and 6 *per Cent. per Ann.*) form'd.

The use of which is plain and easy; for multiplying the figures standing against the number of years required, and under the given rate, by the proposed principal, the product is the amount desired.

Thus,

Thus, if the amount of 40*l.* in 20 years, at 5*l. per Cent. per Annum*, were required, 2,65329, the tabular number, multiply'd by 40*l.* the principal sum, gives 106,13160, or 106*l.* 2*s.* 7*d.* $\frac{1}{2}$, the *Answer*. And so of any other.

2*d.* To find the amount of any annuity, or yearly pension, forborn any number of years whatsoever, at any rate of compound interest.

R U L E.

Multiply the first yearly payment by the amount of 1*l.* for 1 year, and to the product add the second yearly payment, the sum is the amount in 2 years; which, multiply'd again by the said amount, the product, with the addition of the third yearly payment, is the amount for 3 years, &c.

E X A M P L E.

What will a pension of 30*l. per Annum* amount to, being forborn 4 years, at 5*l. per Cent. per Annum*?

	l.	
First yearly payment	30	
multiply'd by	1,05	
	31,50	
Second yearly payment added	30	
	61,50	
		gives 61,50 the amount in 2 years.
	1,05	
	64,5750	
Third yearly payment	30	
	94,5750	
		amount in 3 years.
	1,05	
	99,303750	
Fourth yearly payment	30	
	129,303750	
		amount in 4 years.
		And

And after this manner is the second interest table (which shews the amount of 1 l. annuity for any number of years under 33, at the rates of 5 and 6 l. per Cent. per Ann. compound Interest) compos'd.

But this second table may more easily be form'd from the first, thus: the first line of this second table, or first year's amount being known, add to it the first line of the first table, the sum is the amount of 2 years, or the second line of this table; to which again adding the second line of the first table, you have the third of this, or the amount for three years, &c.

E X A M P L E, at 6 l. per Cent.

<i>I.</i>	
1,	———— First year of the second table.
1,06	———— First of the first table.
2,06	———— Second year of the second table.
1,1236	———— Second year of the first.
3,1836	———— Third year of the second.
1,19101	———— Third year of the first.
4,37461	———— Fourth year of the second, &c.

The use of this second table is in the same manner as the first; as suppose the foregoing question for an example.

E X A M P L E.

$$\begin{array}{rcl}
 4,31012 & \left\{ \begin{array}{l} \text{the amount of 1 l. annuity for} \\ \text{4 years, at 5 l. per Cent.} \end{array} \right. \\
 \text{multiply'd by} & 30 & \text{the yearly sum,} \\
 \hline
 \text{gives} & 129,30360 & \text{as before.}
 \end{array}$$

REBATE,

REBATE, or DISCOMPT.

REBATE, according to simple interest, having been sufficiently explained in *Vulgar Arithmetic*, I shall only here set down one example work'd decimally the common way. And, to complete the section, add Mr. Hatton's new method of finding the present worth and discompt of money, as delivered in his *System of Arithmetic*, pag. 188.

EXAMPLE.

What discompt must be allowed on a bill of 500 *l.* paid 20 days before it is due, rebate at 5 *l. per Cent. per Annum*? And what present money must be paid?

The common way.

Days	l.	Days.
If 365	— 5 —	20
		5
		365)100,0(27397
		2700
		1450
		3550
		2650
		.95

Then

[199]

Then for the discompt.

If 100,27397 ———,27397 ——— 500
500

100,27397) 136,98500 (1,3661 *discompt.*

36711030

66288390

61240080

10756980

• 629593

Or for the present worth.

If 100,27397 ——— 100 ——— 500
500

100,27397) 50000,000 (498,6338 *present worth.*

98904120

86575470

63562940

33985580

39033890

89516990

9297814

Mr. HATTON's New Method.

1. For the *present worth*, multiply the days in a year, the principal given, and 100 into each other, for a dividend; and add the product of 365 by 100 to that of the days multiplied into the rate given for the divisor; so the quotient arising is the answer.

Thus

[200]

Thus in the foregoing example, $365 \times 500 \times 100 = 18250000$ for the dividend, and $365 \times 100 + 20 \times 5 = 36600$ for the divisor.

See the work.

$366 \overline{) 00} 182500 \overline{) 00} (498,6338 \text{ present worth.}$

3610

3160

2320

1240

1420

322

2. For the *discompt*, multiply the rate principal, and days given together for a dividend; and proceeding as before directed for a divisor, the quotient will be the answer.

Thus keeping still the same example, $5 \times 500 \times 20 = 50000$ for the dividend, and $365 \times 100 + 20 \times 5 = 36600$ for a divisor, as before.

See the work.

$366 \overline{) 00} 500 \overline{) 00} (1,3661 \text{ discompt.}$

1340

2240

2240

440

74

The

The truth of both these methods appears, not only by their agreement, but also by the addition of the present worth, and discompt, found ; which, as near as can be, makes the principal.

Thus 498,6338 the present worth,
with 1,3661 the discompt,

makes 499,9999 = 500 l. the given principal.

REBATE at COMPOUND INTEREST.

1st. **T**O find the present worth of any sum to be paid at any number of years to come, rebate being made at any rate of compound interest.

R U L E.

Divide the principal continually so many times by the amount of 1 l. for 1 year, as are the number of years proposed, and the last quotient is the answer.

E X A M P L E.

What is the present worth of 1 l. to be paid at the end of 6 years, rebate at 6 l. per Cent. per Ann. compound interest ?

l.		Years.	
The principal 1,			
,943396	divided by	,943396	present worth at the end of
,889997		,889997	
,839619		,839619	
,792094		,792094	
,747258		,747258	
	gives	,704960	

And thus is the third table made, which shews the present worth of 1 l. due at any number of years to come, under 33, rebate at 5 and 6 l. per Cent. per Annum, compound interest ; the use of which is as the

the other tables, only by multiplying the present worth of 1 *l.* by the given principal, and the product is the present worth required.

Thus, suppose in the example above, the principal had been 200 *l.* the present worth of 1 *l.* viz. .704960 multiply'd by 200, gives 140,992000, the present worth of 200 *l.* and so any other.

2d. To find the present worth of any annuity, or yearly pension, to continue any number of years; rebate being made at any rate, *per Cent. per Annum*, compound interest.

R U L E.

Find by the foregoing rule the present worth of the propos'd annuity, for 1, 2, 3, or so many years as are demanded; and the sum of those respective present worths will be the value of the annuity; that is, the first and second of those values, added together, will be the present worth for two years; and the first, second, and third, added for three years, &c.

E X A M P L E.

What's the present worth of an annuity of 50 *l.* to continue four years, rebate at 5 *l. per Cent. per Annum*?

$$\begin{array}{l}
 1. \\
 50, \\
 47,61904 \left\{ \begin{array}{l} \text{divided by } 1,05 \\ \text{gives,} \end{array} \right. \left\{ \begin{array}{l} 47,61904 \\ 45,35147 \\ 43,19187 \\ 41,13512 \end{array} \right.
 \end{array}$$

The total of which is 177,29750 the present
(worth requir'd.

And after this manner is made table the fourth, which shews the present worth of 1 *l.* annuity, to continue any number of years under 33, rebate at 5
and

1 2 3 4 5 6 7		8 9 10 11 12 13 14	
15 16 17 18 19 20 21		22 23 24 25 26 27 28	
29 30 31 32 33 34 35		36 37 38 39 40 41 42	
43 44 45 46 47 48 49		50 51 52 53 54 55 56	
57 58 59 60 61 62 63		64 65 66 67 68 69 70	
71 72 73 74 75 76 77		78 79 80 81 82 83 84	
85 86 87 88 89 90 91		92 93 94 95 96 97 98	
99 100		101 102	



Decimal Tables,

Shewing

TABLE 1. *The Amount of 1 £. for any number of Years under 33 at the Rates of 3 & 4 per Cent. p An. Com. Int.*

TABLE 2. *The Amount of 1 £. Annuity for any number of Years under 33 at the Rates of 3 & 4 per Cent. p An. C. Int.*

3 Rates 4		Years	3 Rates 4	
1,03000	1,04000	1	1,00000	1,00000
1,06090	1,08160	2	2,03000	2,04000
1,09273	1,12486	3	3,09090	3,12160
1,12551	1,16986	4	4,18363	4,24646
1,15927	1,21665	5	5,30914	5,41632
1,19405	1,26532	6	6,46841	6,63297
1,22987	1,31593	7	7,66246	7,89829
1,26677	1,36857	8	8,89233	9,21423
1,30477	1,42331	9	10,15910	10,58279
1,34392	1,48024	10	11,46388	12,00610
1,38423	1,53945	11	12,80780	13,48635
1,42576	1,60103	12	14,19203	15,02580
1,46853	1,66507	13	15,61780	16,62684
1,51259	1,73167	14	17,08632	18,29191
1,55797	1,80094	15	18,59891	20,02359
1,60471	1,87298	16	20,15688	21,82453
1,65285	1,94790	17	21,76159	23,69751
1,70243	2,02582	18	23,41443	25,64541
1,75350	2,10685	19	25,11687	27,67123
1,80611	2,19112	20	26,87037	29,77808
1,86029	2,27877	21	28,67649	31,96920
1,91610	2,36992	22	30,53678	34,24797
1,97359	2,46472	23	32,45288	36,61789
2,03279	2,56330	24	34,42647	39,08260
2,09378	2,66583	25	36,45926	41,64591
2,15659	2,77247	26	38,55304	44,31174
2,22129	2,88337	27	40,70963	47,08421
2,28793	2,99870	28	42,93092	49,96758
2,35656	3,11865	29	45,21885	52,96629
2,42726	3,24340	30	47,57542	56,08494
2,50008	3,37313	31	50,00268	59,32833
2,57508	3,50806	32	52,50276	62,70147

Webster script

Bickham sculp

Decimal Tables,

Shewing

TABLE 3. *The present Worth of 1 £ due at any Number of Years under 33. Rebate at 3 & 4 per Cent. Ann. Com. Int.*

TABLE 4. *The present Worth of 1 £ Annuity for any Number of Years under 33. Rebate at 3 & 4 per Cent. Ann.*

3 Rates 4		Years	3 Rates 4	
.970874	.961538	1	0,97087	0,96154
.942596	.924556	2	1,91347	1,88609
.915142	.888996	3	2,82861	2,77509
.888487	.854804	4	3,71710	3,62989
.862609	.821927	5	4,57971	4,45182
.837484	.790314	6	5,41719	5,24214
.813091	.759918	7	6,23028	6,00205
.789409	.730690	8	7,01969	6,73274
.766416	.702587	9	7,78611	7,43533
.744094	.675564	10	8,53020	8,11090
.722421	.649581	11	9,25262	8,76048
.701380	.624597	12	9,95400	9,38507
.680951	.600574	13	10,63496	9,98565
.661118	.577475	14	11,29607	10,56312
.641862	.555264	15	11,93793	11,11839
.623167	.533908	16	12,56110	11,65230
.605016	.513373	17	13,16612	12,16567
.587394	.493628	18	13,75351	12,65930
.570286	.474642	19	14,32380	13,13394
.553676	.456387	20	14,87747	13,59033
.537550	.438833	21	15,41502	14,02916
.521892	.421955	22	15,93692	14,45112
.506692	.405726	23	16,44361	14,85684
.491934	.390121	24	16,93554	15,24696
.477606	.375117	25	17,41315	15,62208
.463695	.360689	26	17,87684	15,98277
.450189	.346817	27	18,32703	16,32959
.437077	.333477	28	18,76411	16,66306
.424346	.320651	29	19,18845	16,98371
.411987	.308319	30	19,60044	17,29203
.399987	.296460	31	20,00043	17,58849
.388337	.285058	32	20,38877	17,87355



1875



Decimal Tables

shewing

TABLE 1.st The Amount of one Pound for any Number of Years under 33, at y^e Rates of 5 & 6 p^{ct}. p^{an}. Com. Inter.

TABLE 2.^d The Amount of 1st Annuity for any number of years under 33, at y^e Rates of 5 & 6 p^{ct}. p^{an}. Com. Interest.

5	6	Years	5	6
1,05000	1,06000	1	1,00000	1,00000
1,10250	1,12360	2	2,06000	2,06000
1,15762	1,19101	3	3,15250	3,18360
1,21550	1,26247	4	4,31012	4,37461
1,27628	1,33822	5	5,52563	5,63709
1,34009	1,41852	6	6,80191	6,97532
1,40710	1,50363	7	8,14200	8,39383
1,47745	1,59384	8	9,54910	9,89746
1,55132	1,68948	9	11,02656	11,49131
1,62889	1,79084	10	12,57789	13,18079
1,71034	1,89829	11	14,20678	14,97164
1,79585	2,01219	12	15,91712	16,86994
1,88565	2,13292	13	17,71298	18,88213
1,97993	2,26090	14	19,59863	21,01506
2,07892	2,39655	15	21,57856	23,27597
2,18287	2,54035	16	23,65749	25,67252
2,29201	2,69277	17	25,84036	28,21288
2,40662	2,85434	18	28,13238	30,90565
2,52695	3,02559	19	30,53900	33,75999
2,65329	3,20713	20	33,06595	36,78559
2,78596	3,39956	21	35,71925	39,99272
2,92526	3,60353	22	38,50521	43,39229
3,07152	3,81975	23	41,43047	46,99582
3,22510	4,04893	24	44,50199	50,81557
3,38635	4,29187	25	47,72709	54,86451
3,55567	4,54938	26	51,11345	59,15638
3,73345	4,82234	27	54,66912	63,70576
3,92013	5,11168	28	58,40258	68,52811
4,11613	5,41838	29	62,32271	73,63979
4,32194	5,74349	30	66,43884	79,05818
4,53804	6,08810	31	70,76079	84,80167
4,76494	6,45338	32	75,29883	90,88977

Webster Script.

Bickham Script.

Decimal Tables

shewing

TABLE 3. The present Worth of 1^l. due at any number of years to come under 33. Rate at 5 & 6 p^{ct}. p^{an}. Com. Interest
TABLE 4. The present Worth of 1^l. Annuity to continue any number of years under 33. Rate at 5 & 6 p^{ct}. p^{an}. Com. Interest

Rates		years	Rates	
5	6		5	6
.952381	.943396	1	0.95238	0.94339
.907030	.889996	2	1.85941	1.83333
.863838	.839619	3	2.72324	2.67301
.822702	.792093	4	3.54595	3.46510
.783526	.747258	5	4.32947	4.21236
.746215	.704960	6	5.07569	4.91732
.710682	.665057	7	5.78637	5.58238
.676839	.627412	8	6.46321	6.20979
.644609	.591898	9	7.10782	6.80169
.613913	.558394	10	7.72173	7.36008
.584679	.526787	11	8.30641	7.88687
.556837	.496969	12	8.86325	8.38384
.530321	.468839	13	9.39357	8.85268
.505068	.442301	14	9.89864	9.29498
.481017	.417265	15	10.37965	9.71225
.458111	.393647	16	10.83777	10.10589
.436296	.371364	17	11.27406	10.47726
.415520	.350343	18	11.68958	10.82760
.395734	.330513	19	12.08532	11.15811
.376889	.311804	20	12.46221	11.46992
.358942	.294155	21	12.82115	11.76407
.341849	.277505	22	13.16300	12.04158
.325571	.261797	23	13.48857	12.30338
.310067	.246978	24	13.79864	12.55035
.295302	.232998	25	14.09394	12.78335
.281240	.219810	26	14.37518	13.00316
.267848	.207368	27	14.64303	13.21053
.255093	.195630	28	14.89812	13.40616
.242946	.184536	29	15.14107	13.59072
.231377	.174110	30	15.37245	13.76483
.220369	.164254	31	15.59281	13.92908
.209865	.154956	32	15.80267	14.08404



and 6 l. *per Cent. per Ann.* compound interest. Which table, as the others, will give the present worth of any sum whatsoever, by only multiplying the value of 1 l. by the sum proposed. Thus, suppose the foregoing example.

E X A M P L E.

3,54595 *the present worth of 1 l. an. for 4 years.*
multiply'd by 50 *the annuity proposed,*

gives 177,29750 *as before.*

The fourth table may be also form'd from the third, thus :

The first year is the same in both ; the first and second years of the third, added together, make the second of the fourth ; the second year of the fourth, added to the third year of the third, makes the third of the fourth table, &c. See the example.

E X A M P L E, at 6 l. per Cent.

,94339 * *The first year of both tables.*

,88999 *The second year of the third table.*

1,83339 *The second year of the fourth.*

,83961 *The third year of the third.*

2,67300 *The third year of the fourth.*

&c.

* *Note, 1 is added to the right hand figure, in consideration of the places omitted.*

More

*More E X A M P L E S for the Exercise of the foregoing
RULES and TABLES.*

Ex. 1st. What will 1000*l.* increase to, forborn 22 years, at 5 *per Cent. per Annum* compound interest?

Answer, 2925*l.* 5 *s.* 2 *d.* $\frac{1}{2}$.

Ex. 2^d. What would an annuity of 520*l.* forborn 17 years, at 6 *per Cent.* compound interest, amount to?

Answer, 14670*l.* 13 *s.* 11 *d.* $\frac{1}{4}$.

Ex. 3^d. What present money would discharge a debt of 7500*l.* to be paid at the end of 10 years, rebate being made at 5 *l. per Cent. per Annum*?

Answer, 4604*l.* 6 *s.* 11 *d.* $\frac{1}{4}$.

Ex. 4th. What is the yearly rent of 65*l.* to continue 30 years, worth in ready money, rebate being made at 6 *per Cent. per Annum* compound interest?

Answer, 894*l.* 14 *s.* 3 *d.* $\frac{1}{4}$.

Ex. 5th. What is the present worth of a reversion of a lease of 500 *l. per Annum*, to continue 20 years, but not to commence till after the end of five years, allowing the purchaser 6 *per Cent. per Annum* compound interest?

Note, To work questions of this nature, there must be two distinct operations, *viz.*

1st. The present worth of the proposed annuity; for the given time of its continuance must be found as if it were immediately to commence. Then,

2^d. See what principal, forborn at the given interest, would, in the time to the commencing of the reversion, amount to the aforesaid present worth, and that principal will be the present worth of such annuity in reversion.

Thus

Thus, by table the 4th, the present worth of 1 l. per Annum, to continue 20 years, at 6 l. per Cent. is found to be

11,46992	
which, multiplied by the proposed annuity,	500
	gives 5734,9600

Then, in the first table, against 5 years, the time to the commencing of the reversion, under the same rate of interest, is found 1,33822, the amount of 1 l. in the said time; therefore,

If 1,33822 arise from 1 l. from what principal does 5734,96000 arise?

Answer, $4285,5135 = 4285 \text{ l. } 10 \text{ s. } 3 \text{ d. } \frac{1}{4}$.

Or it may more easily be done by first finding the present worth of the annuity for the whole time, both of possession and reversion, and then deducting the present worth of the possession; the remainder will be the present worth of the reversion. Thus 12,78335, the tabular number against 25 years, the whole time being multiply'd by 500, gives 6391,67500; from which subtracting 2106,18000, the produce of 4,21236, the tabular number against 5 years, the time of the possession, the remainder 4285,49500, is the value of the reversion: Within a trifle the same answer as before.

FREEHOLD ESTATES.

TO find the present worth of any annual rent, to continue for ever, commonly called fee-simple.

K

RULE.

R U L E.

Divide the proposed rent by the interest of 1 *l.* for one year, (which at 5 *l. per Cent.* is ,05, or at 6 *per Cent.* ,06, as before shewn) and the quotient is the present value of the estate.

E X A M P L E.

What is an estate of 200 *l. per Annum*, to continue for ever, worth in ready money, allowing the purchaser 5 *l. per Cent. per Annum*, compound interest?

$$\begin{array}{r} \text{—,05)200,00(} \\ \text{Answer, 4000 } l. \end{array}$$

Or it may be done by multiplying the fee-simple of 1 *l.* by the yearly rent proposed.

The fee-simple of 1 *l. per Annum* compound interest, at

1 per Cent.	2 per Cent.	3 per Cent.	4 per Cent.	5 per Cent.
100,00000	50,0000	33,33333	25,00000	20,00000
6 per Cent.	7 per Cent.	8 per Cent.	9 per Cent.	10 per Cent.
16,66667*	14,28571	12,50000	11,11111	10,00000

* *Note*, In contracting a line of decimals from many to fewer places, 1 is added to the last figure retained, if the next omitted figure exceedeth 5; which rule hath been all along observed in the tables, &c.

The foregoing question work'd by the last rule.

	<i>l.</i>
The fee-simple of 1 <i>l. per Ann.</i> at 5 <i>l. per Cent.</i> is	20
which, multiplied by the yearly rent,	200
gives, as before,	4000

I will

I will now only add another example or two, for the reader's exercise, and conclude this rule.

1. *A.* has the possession of an estate of 130 *l.* *per Annum*, to continue 20 years; *B.* has the reversion of the same, from that time for ever. What must *A.* give *B.* if he would purchase his reversion? And what must *B.* give *A.* if he would buy his possession, accounting 6 *per Cent.* compound interest in each case?

	<i>l.</i>	<i>s.</i>	<i>d.</i>
<i>Answer</i> , { <i>A's</i> possession is worth	149	1	9 $\frac{1}{2}$
{ <i>B's</i> reversion is worth	675	11	6 $\frac{1}{2}$

2. *A.* agrees with *B.* for an annuity of 600 *l.* *per Annum*, to continue 25 years, to give him the present worth of it, at 6 *l.* *per Cent. per Annum*; but not having money enough by him, offers to make over to him a freehold estate of 12 *l.* *per Annum*, at the same interest: What money besides will pay his purchase?

Answer, 7470 *l.* 0 *s.* 2 *d.* $\frac{1}{2}$.

C H A P. VI.

Of F E L L O W S H I P.

Without taking any notice of the common way of working this rule, observe this much more compendious method.

R U L E.

Divide the whole gain, or loss, by the whole stock, and multiply the quotient by each man's particular stock; the several products are the respective gains of each.

Note, It is necessary to place so many cyphers on the right hand of your dividend, as will bring out six or seven fractional places in the quotient: but if the last, or any figure of it will recur infinitely, put down the vulgar fraction in that place; and, to make up the several products, do as shewn in the indirect Rule of Three.

E X A M P L E.

Suppose *A. B. and C.* trading together, *A* puts in 500 *l.* *B.* 700 *l.* and *C.* 1200 *l.* the whole gain is 836 *l.* What share of it belongs to each?

$$\left. \begin{array}{l} A=500 \\ B=700 \\ C=1200 \end{array} \right\} = 2400) 836.0000 (.348\frac{1}{3} = .348\frac{1}{3}$$

$$\begin{array}{r} .348\frac{1}{3} \\ \hline 500 \end{array}$$

$$\begin{array}{r} .348\frac{1}{3} \\ \hline 700 \end{array}$$

$$\begin{array}{r} .348\frac{1}{3} \\ \hline 1200 \end{array}$$

$$\left. \begin{array}{l} A's \text{ share} = 174.1\frac{2}{3} \\ B's \text{ . . .} = 243.8\frac{1}{3} \\ C's \text{ . . .} = 418.0 \end{array} \right\} = 836 \text{ l. the total gain.}$$

See

See another *EXAMPLE* with time.

A. puts into company 525 *l.* 10 *s.* for six months,
B. 382 *l.* 15 *s.* for eight months, and C. 1000 *l.* for
four months; they gain in all 1286 *l.* 12 *s.* How much
is that for each?

	525,5 6	382,75 8	1000 4
A's stock	3153,0	3062,00	4000
B's—	3062		
C's—	4000		
	10215)1286,600000(,125952		
	,125952	,125952	,125952
	3153	3062	4000
A's gain	397,126656	385,665024	503,808000
B's—	385,665024		
C's—	503,808000		

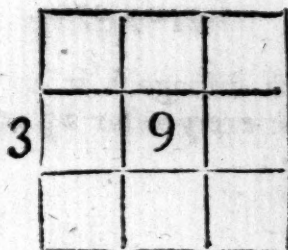
Total, 1286,599680, or 1286 *l.* 12 *s.* nearly.

C H A P. VII.

Of EVOLUTION, or EXTRACTION
of ROOTS.*The SQUARE ROOT.*

EXTRACTION of the SQUARE ROOT is finding the side of a square figure ; or, numerically speaking, it is finding what number, multiplied by itself, will produce the number given. Thus the square root of 16 is 4, 4 times 4 making 16.

What a square is, may be seen in the following figure ; which being divided every way into 3 equal parts, its whole content or square is 9, and its side or root is 3.



Square numbers are either single or compound.

A single square number is always less than 100, being produced by the multiplication of some one single figure by itself; as 16 from 4, &c. So that the root of any single square may be found in the annexed table, always taking the root of the next less square, for any number not there inserted; as, for 26 take 5, or 3 for 10, &c.

Square	1	4	9	16	25	36	49	64	81
Roots	1	2	3	4	5	6	7	8	9

A compound square number being compos'd by the multiplication of two or more figures by themselves, always exceeds 100; as 144, which is 12 times 12; or 195, which is 15 times 15, &c.

If the root therefore is express'd by two figures, its square must at least consist of three; for the least root express'd by two figures is 10, whose square is 100. And if the root has three figures, its square must at least have five. If four, the square has seven, &c. So that you cannot augment the root one figure, but you increase the square two.

To find the Root of any compound square Number; as suppose 2704.

1st. You must distinguish it into single squares, by placing dots over every other figure, beginning at the right hand, thus;

. .
2704

And so many dots as happen, so many places will the root consist of; which in this example are two.

K 4

2dly.

2^{dly}. Drawing a crooked line on the right hand of your number, as in division, find the root of your first single square, and place it in the quotient.

See the work.

$$\begin{array}{r} \cdot \cdot \\ 2704(5 \end{array}$$

3^{dly}. Placing the square of the root found, under the first single square, subtract, and set down the remainder; bringing down to it the next single square, call the line a *Resolvend*.

$$\begin{array}{r} \cdot \cdot \\ 1704(5 \\ 25 \\ \hline \end{array}$$

• 204 *Resolvend*.

4^{thly}. Drawing another crooked line on the left hand of the said resolvend, place beyond it the double of the quotient, in form of a divisor.

$$\begin{array}{r} \cdot \cdot \\ 2704(5 \\ 25 \\ \hline \end{array}$$

10).204

5^{thly}. Dividing the resolvend, all but the unit's place, by the said divisor (which, in this example, is the 10's in 20) set down the number of times it goes (*viz.* 2) both in the quotient, and on the right hand of the divisor.

$$\begin{array}{r} \cdot \cdot \\ 2704(52 \\ 25 \\ \hline \end{array}$$

102)204 *Resolvend*.

6^{thly}.

6thly. Multiplying the whole divisor by the figure last placed in the quotient, set down the product under, and subtract it from the resolvend.

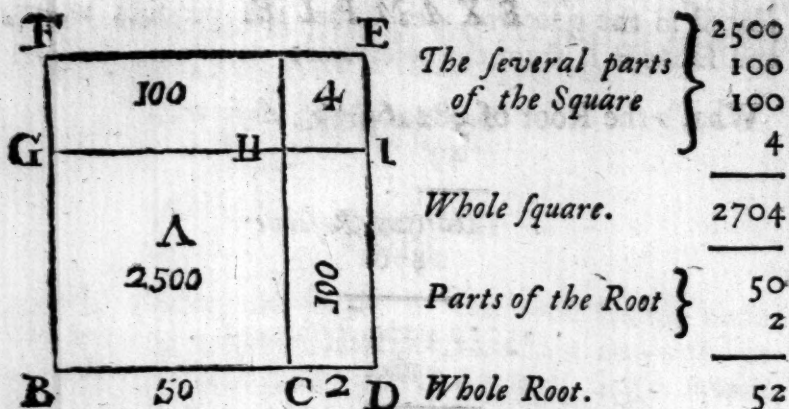
See the work :

$$\begin{array}{r}
 2704 \overline{)52} \\
 25 \\
 \hline
 102 204 \\
 204 \\
 \hline
 0
 \end{array}$$

Note, If the divisor, multiply'd by the quotient-figure, gives a product greater than the resolvend, 'tis false, and must be rectified by a smaller quotient-figure.

Extraction of the square root being a matter rather lineal than numeral, the reason of the rules above given for its operation, will best appear from the following figure: Wherein let it be considered, that if the root *B. C.* of the square *A*, be augmented by the magnitude *C. D.* the said square will be enlarged by the figure *C. D. E. F. G. H. I.* equal to an oblong or long square, whose length shall be the root *B. C.* twice taken, with the line *C. D.* and breadth the same line *C. D.*

N. B. The particular contents of the several parts of the figure are as therein set down.



In the operation therefore, having taken the nearest root of the first square 27, *viz.* 5. and placed it in the quotient, from the place it stands in (*viz.* the second) it must be accounted 50; the square of which, *viz.* 2500 (marked *A.* in the figure) being subtracted from the given number 2704, there remains 204, equal to the figure *C. D. E. F. G. H. I.* In order to find the content of which, I double the root *B. C.* *viz.* 50, for the two bases *C. H.* and *H. G.* which making 100, I set it down as a divisor; and to find the height of the said figure, *viz.* *C. D.* I seek how often the said divisor is contain'd in the number 204; and finding it twice, I set down two in the quotient for the said height, and also in the unit's place of my divisor for the base *H. I.* Then multiplying the bases by the height, I have the content of the whole annexed figure *C. D. E. F. G. H. I.* *viz.* 204, exactly equal to the number remaining, after the subtraction of the first great square 2500; so that the number given, *viz.* 2704, appears to be a perfect square, whose root or side is exactly 52.

Note, As many single squares as are in the number proposed, so many times must the work of the three last rules be repeated; every square being to be brought down, as directed in the third rule.

E X A M P L E.

EXAMPLE.

What's the Root of 582169763 *Answer.*

$$\begin{array}{r}
 49 \\
 \hline
 146 \overline{) 921} \text{ *Resolvend.* } \\
 \underline{876} \\
 1523 \overline{) 4569} \text{ *Resolvend.* } \\
 \underline{4569} \\
 0
 \end{array}$$

In this example, to 45 the second remainder, 69 the next square, being brought down, makes 4569 for a new resolvend; and 76, the quotient-figures, being doubled for a new divisor, makes 152, which going three times in the resolvend, 3 is plac'd both in the quotient, and on the right hand of the divisor, which being then multiply'd by 3, gives 4569, equal to the resolvend; so that nothing remains.

P R O O F.

The proof of this extraction, is only multiplying the root found by itself; which, if right, will produce the square number given. *Note,* If any thing remains, it must be taken in.

To find the fractional Part of the Root.

Add to the number given a competent number of pairs of cyphers, and proceed by the foregoing rules: As suppose 75896 were given for an example.

EXAMPLE.

$$\begin{array}{r}
 75896,000000(275,492 \\
 \underline{4} \\
 47)358 \\
 \underline{329} \\
 545)2996 \\
 \underline{2725} \\
 5504)27100 \\
 \underline{22016} \\
 55089)508400 \\
 \underline{495801} \\
 550982)1259900 \\
 \underline{1101964} \\
 \cdot 157936
 \end{array}$$

Note, As many pairs of cyphers as are added, so many fraction places of decimal parts will be in the root.

To extract the SQUARE ROOT of a VULGAR FRACTION.

FIND the root of the numerator for a numerator; and let the root of the denominator be its denominator; thus the square root of $\frac{16}{25}$ is $\frac{4}{5}$, and of $\frac{4}{9}$ is $\frac{2}{3}$, &c.

The root of a mix'd number is also found after the same manner, being first reduced into an improper fraction;

fraction; thus $6\frac{1}{4}$, reduced, makes $2\frac{5}{4}$, the root of which is $\frac{5}{2}$, or $2\frac{1}{2}$.

But if either a proper fraction, or a mix'd number, be incommensurable to its root; to extract it, reduce the fraction to a decimal of an even number of places, and extract the root as if it were a whole number: thus $\frac{7}{8}$, reduced to a decimal, will be, 8750, the square root of which is, 935, &c. but had it been $4\frac{7}{8}$, which reduced is 4,8750, the root would be 2,20794, &c.

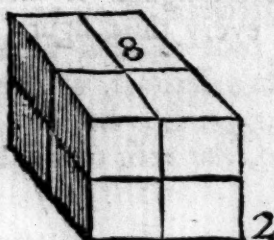
The CUBE ROOT.

EXtraction of the CUBE ROOT, is finding the side of a solid figure, whose length, breadth, and depth, are equal; or, numerically speaking, it is finding what number multiply'd into itself, and then into the product, will produce the number given; thus the Cube Root of 64 is 4, 4 times 4 being 16 and 4 times 16 make 64.

What a Cube is, may be something explain'd by the following figure; wherein 8 dice are so disposed, that there are 2 every way, that is, 2 in length, 2 in breadth, and 2 in height; so that 8 is the cube, and 2 the root.

[218]

See the figure.



Cube-numbers are either single or compound.

A single cube-number is always less than 1000, being produced by the multiplication of one single figure, first by itself, and then by its product; as 125 from 5, &c. So that the root of any single cube may be found in the annexed table; always remembring to take the root of the next less cube for any number not there inserted, as for 513 take 8, or 6 for 220, &c.

Cubes	1	8	27	64	125	216	343	512	729
Squares	1	4	9	16	25	36	49	64	81
Roots	1	2	3	4	5	6	7	8	9

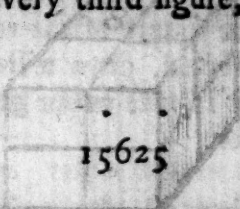
A compound cube-number, being composed by the multiplication of 2 or more figures, first by themselves, and then by their product, always exceeds 1000, as 1728 from 12, or, 15625 from 25, &c.

If therefore the root is express'd by two figures, its cube must at least consist of four; for the least root express'd by two figures is 10, whose cube is 1000; if the root has three figures, its cube must at least have seven, &c. So that you cannot augment the root one figure, but you increase the cube three.

Therefore,

Therefore, to find the root of any compound cube-number, as suppose 15625.

1st. You must distinguish it into single cubes, by placing dots over every third figure, beginning at the right hand, thus :



And so many dots as happen, so many places will the root consist of ; which, in this example, are two.

2^{dly}. Drawing a crooked line on the right hand of your number, as in division, set down as a quotient the root of your first single cube ; which, in this example, is 2.

3^{dly}. Placing 8 the cube of 2, the root found, under 15 the first single cube, subtract, and to the remainder 7 bring down 625, the next single cube, and it will make 7625 ; which call a Resolvend.

See the work.

$$\begin{array}{r} \cdot \cdot \cdot \\ 15625(2 \\ \underline{8} \\ 7625 \text{ Resolvend.} \end{array}$$

4^{thly} Draw a line under the resolvend, and tripling the square of the root 2, set the said triple square (*viz.* 12) under the resolvend ; so that units in the said triple square may stand under the place of hundreds in the resolvend.

5^{thly}.

5^{thly}. Subscribe also the triple of the root 2, (*viz.* 6) so that units, in this, may stand under the place of tens in the resolvend.

6^{thly}. The triple square of the root, and triple root, being placed as directed, draw a line under them, and add them together in the order they are placed; the sum 126 is a divisor.

7^{thly}. Accounting all the resolvend, except the place of units, (*viz.* 762) a dividend, seek how often the divisor (126) is contained in it, and place the number of times (which is here 5) in the quotient.

8^{thly}. Draw a line under the divisor, and multiplying the triple square (12) by 5, the figure last placed in the quotient, set the product (60) so under the said triple square, that units may stand under units, and tens under tens.

9^{thly}. Squaring the figure (5) last placed in the quotient, multiply its square (*viz.* 25) by the triple root (6), and place the product (150) so, that units, in this, may stand under units in the said triple number.

10^{thly}. Subscribe the cube of (5), the figure last placed in the quotient, which will be 125; so that tens, in this, may stand under units in the former product.

11^{thly}. Then drawing a line, add the three numbers last placed together; and subtracting the sum 7625 (which is called the *ablatitium*) from the resolvend, set down the remainder (if any) in order underneath, as in common subtraction.

See the work.

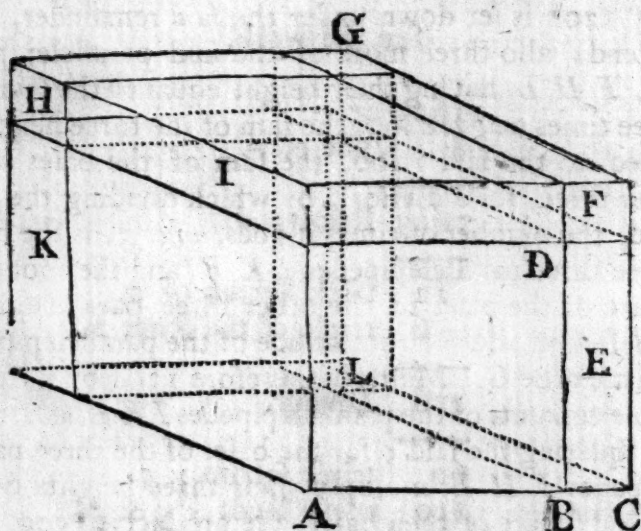
$$\begin{array}{r}
 15625(25 \\
 8 \quad \text{Cube of 2.} \\
 \hline
 7625 \quad \text{Resolvend.} \\
 \hline
 12 \quad \text{triple square of 2.} \\
 6 \quad \text{triple of the Root 2.} \\
 \hline
 126 \quad \text{Divisor.} \\
 \hline
 60 \quad \text{triple square} \times 5. \\
 150 \quad \text{triple Root} \times 5 \times 5^d \\
 125 \quad \text{Cube of 5.} \\
 \hline
 7625 \quad \text{Ablatitium,} \\
 \hline
 \dots 0
 \end{array}$$

Note, If the given number hath more places, the next single cube must be brought down to the last remainder, for a new resolvend; and the work of the 4th, 5th, 6th, 7th, 8th, 9th, 10th, and 11th rules, must be repeated as often as you so form a new resolvend.

Note also, If the *ablatitium* is greater than the resolvend, the work is false, and must be rectified by placing a lesser figure in the quotient.

To account for these rules of extracting the cube root, we must again make use of a figure with literal references, as follows:

From



From whence it appears, That if the root $A. B.$ of the cube $A. D.$ be augmented by the magnitude $B. C.$ the cube is thereby enlarged by the addition of seven parallelepipeds, * viz. 1st. The little cube $G.$ whose root is the line $B. C.$ 2^d. The three parallelepipeds, mark'd $I. K. E.$ which have the square of the root $A. B.$ for base, and the line $B. C.$ for height. 3^d. The three parallelepipeds mark'd $F. H. L.$ which have the square of the line $B. C.$ for base, and the root $A. B.$ for height.

In the operation therefore of the foregoing example, 15625 being divided into single cubes, it appears that the root will consist of two places: The nearest root then of the first cube 15, viz. 2, being placed in the quotient, where its value will be 20, from the place it stands in, and its cube 8000 subtracted from the given number 15625, the remainder 7625 will be equal to the seven parallelepipeds, $G. I. K. E. F. H. L.$ the bases of three of which, viz. $I. K. E.$ being the square of the root $A. B.$ three times the square of 20, viz.

* Parallelepipeds is a geometrical term, signifying regular figures, whose opposite sides and angles are equal.

viz. 1200 is set down under the said remainder, or resolvend; also three more of the said parallelepipeds, *viz.* *F. H. L.* having their height equal to the said root, three times 20, *viz.* 60, the sum of the three heights, is added to the said 1200, the sum of the bases of the other three, for a divisor; by which dividing the resolvend, the number of times it goes, *viz.* 5, is the height of the three parallelepipeds *I. K. E.* and the root of the square of the bases of the other three parallelepipeds *F. H. L.* and also the root or side of the parallelepipedon, or little cube *G.* Multiply therefore 1200 by 5, for the whole contents of the parallelepipeds *I. K. E.* *viz.* 6000; and squaring the said 5 for the bases of the three parallelepipeds *F. H. L.* multiply their three heights 60, by the said square of their bases, the product is 1500, their whole contents; under which setting down also the cube of 5, *viz.* 125, for the content of the parallelepipedon *G.* the sum of these three numbers I find to be just 7652, exactly equal to the resolvend; which shews the number given to be a perfect cube, whose root is exactly 25.

To make all more plain, see the work at length, with the vacant places fill'd with cyphers.

15625	20
8000 Cube A. D.	5
7625 Remainder, or Resolvend.	25 Root.
1200 Bases of Parallelepipeds I. K. E.	
60 Height of Parallelepipeds F. H. L.	
1260 Divisor.	
6000 Content of Parallelepipeds I. K. E.	
1500 Content of Parallelepipeds F. H. L.	
125 Content of Parallelepipedon G.	
7625 Total, or Ablatitium.	

PROOF.

P R O O F.

The proof of this extraction is, by multiplying the root found, first by itself, and that product again by the root ; adding the remainder, if there be any, to the last product : thus,

$$\begin{array}{r}
 \text{multiply'd by} \quad 25 \text{ the Root.} \\
 \quad \quad \quad 25 \\
 \hline
 \quad \quad \quad 125 \\
 \quad \quad \quad 50 \\
 \hline
 \text{gives} \quad 625 \\
 \text{again by} \quad 25 \text{ the Root.} \\
 \hline
 \quad \quad \quad 3125 \\
 \quad \quad \quad 1250 \\
 \hline
 \text{gives} \quad 15625
 \end{array}$$

To find the fractional part of the Root.

Add to the number given, so many ternaries of cyphers, as you would have places of decimal parts in the root, and proceed by the foregoing rules. As suppose 865635 were given for an example.

EXAMPLE.

EXAMPLE.

865	635	000	000	000	(95,304 Root.
729					
136	635				1st Resolvend.
24	3				
	27				
24	57				Divisor.
121	5				
6	75				
	125				
128	375				Ablatitium.
8	260	000			2d Resolvend.
2	707	5			
	2	85			
2	710	35			Divisor.
8	122	5			
	25	65			
		27			
8	148	177			Ablatitium.
	111	823	000		3d Resolvend.
	272	462	7		
		28	59		
					Divisor.
	272	491	29		
	111	823	000		4th Resolvend.
	27	246	270	0	
			285	90	
	27	246	555	90	Divisor.
	108	985	080	0	
		4	574	40	
				64	
	108	989	654	464	Ablatitium.
	2	833	345	536	Remainder.

In

In this example, there being three ternaries of cyphers, the root must have three fractional places.

Note, The divisor going no times in the third resolvend, the same figures are immediately brought down again, and with a new ternary of cyphers form the 4th resolvend.

To extract the Cube Root of a vulgar Fraction.

Find the root of the numerator given for a new numerator, and the root of the denominator given for a new denominator; thus the cube root of $\frac{64}{125}$ is $\frac{4}{5}$.

But if the vulgar fraction be incommensurable to its root, reduce it to a decimal of three, six, nine, or any ternary of places, and then extract its root.

CONTENTS.

VULGAR ARITHMETIC.

	Page
Chap. I. { Numeration,	2
{ Addition,	3
{ Subtraction,	6
{ Multiplication,	7
{ Division.	12
Tables of Coins, Weights, &c.	17
Chap. II. Reduction,	21
Chap. III. The Rule of Three { Direct,	24
{ Indirect,	33
{ Double,	35
Chap. IV. { Practice,	45
{ Trett and Tare,	59
Chap. V. Interest, { Simple,	66
{ Compound,	73
Chap. VI. Rebate, or Discount,	77
Chap. VII. Equation of Payments,	85
Chap. VIII. Fellowship, { without Time,	87
{ with Time,	90
Chap. IX. Alligation, { Medial,	96
{ Alternate,	99
Chap. X. The Rule of False, { Single Position,	106
{ Double Position,	108
Chap. XI. Exchanges with { France,	112
{ Italy,	117
{ Spain,	119
{ Portugal,	122
{ Holland, &c.	124
EXPLANATION of CHARACTERS,	128
Chap.	

CONTENTS.

Chap. XII. <i>Progression,</i>	{	<i>Arithmetical,</i>	
		<i>Geometrical,</i>	133
Chap. XIII. <i>Vulgar Fractions,</i>			141

Decimal Arithmetic.

	{	Numeration,	Page 165
		Addition,	166
Chap. I.		Subtraction,	167
		Multiplication,	168
		Division,	170
Chap. II. <i>Reduction,</i>			175
Decimal Tables of COIN, &c.			182
Chap. III. <i>The Rule of Three</i>	{	Direct,	183
		Indirect,	185
		Double,	186
Chap. IV. <i>Practice,</i>			187
	{	Simple Interest,	190
		Table shewing the Number of Days,	195
		Compound Interest,	195
		The Amount of Annuities,	196
Chap. V.	{	Rebate at Simple Interest,	198
		Rebate at Compound Interest,	201
		The present Worth of Annuities,	203
		Decimal Tables of Interest and Rebate,	203
		Freeholds,	205
Chap. VI. <i>Fellowship,</i>	{	without Time,	208
		with Time,	209
Chap. VII. <i>Extraction of the</i>	{	Square Root,	210
		Cube-Root,	217

F I N I S.